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Analyzing Single- and Two-parameter Models for Describing Oil Recovery in Imbibition from Fractured Reservoirs

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Highlights

- A data bank of different experimental and numerical imbibition recovery curves at various rock and fluid properties were collected.
- The single- and two-parameter models used for dose response modeling, including Weibull, beta-Poisson, and Logit models were examined to describe oil/gas recovery.
- Results show that among the two-parameter models, the Weibull demonstrates better capability for describing imbibition process.

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Abstract

The imbibition process is known as one of the main production mechanisms in fractured reservoirs where oil/gas-filled matrix blocks are surrounded by water-filled fractures. Different forces such as gravity and capillary play a role in production from a fractured reservoir during imbibition and complicate the imbibition process. In previous works, single-parameter models such as the Aronofsky model and Lambert W function were presented to model imbibition recovery from matrix blocks. The Aronofsky model underestimates early time recovery and overestimates late time recovery, and Lambert W function is suitable for water wet cases. In this work, a data bank of different experimental and numerical imbibition recovery curves at various rock and fluid properties were collected. Then, a rigorous analysis was performed on the models utilized to describe oil/gas recovery during the imbibition process. In addition to investigating the single-parameter models, two-parameter models used for dose-response modeling, including Weibull, beta-Poisson, and Logit models were examined. The results of this work demonstrate that using two-parameter models can improve the prediction of imbibition behavior. Moreover, among the two-parameter models, the Weibull has the capability to describe the imbibition process better.

Keywords: Aronofsky Model, Dose-response Models, Lambert W Function, Matrix Block

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1. Introduction

In naturally fractured reservoirs, two media are interacting: fracture and matrix (Ardakany et al., 2014; Mirzaei-Paiaman and Masihi, 2013). The fractures with considerable permeability function as a path for producing hydrocarbon toward wells (Figure 1), but the matrix blocks have much lower permeability and work as a source of hydrocarbons (Jafari et al., 2018 and 2017). When the wetting phase displaces the nonwetting phase in a reservoir, this recovery mechanism is called imbibition, which is known as an important recovery mechanism (Ghaedi et al., 2015; Ghasemi, 2018; Mirzaei-Paiaman and Masihi, 2013; Mirzaei-Paiaman, 2015; Standnes, 2010a; Xie and Morrow, 2001). Moreover, because different forces exist during imbibition process, a precise description of this process faces some limitations (Fries and Dreyer, 2008; Ghaedi and Riazi, 2016; Gupta and Civan, 1994; Kazemi et al., 1992; Li and Horne, 2000; Standnes, 2010b; Xie and Morrow, 2001). One of the major issues is to predict the recovery factor at any time in the imbibition process from a single matrix block (Abbasi et al., 2017; Harimi et al., 2019). Efforts have been made to predict imbibition recovery as a function of time (Abbasi et al., 2018; Ardakany et al., 2014; Aronofsky et al., 1958; Ghaedi and Riazi, 2016; Mirzaei-Paiaman, 2015; Mirzaei-Paiaman et al., 2011; Standnes, 2010b). The Aronofsky model was the first correlation introduced to predict oil recovery during imbibition (Aronofsky et al., 1958). This correlation can be fitted to the given data by tuning one parameter:

$$\frac{RF(t)}{RF^{\max}} = \left(1 - e^{-at}\right) \tag{1}$$

where *RF* is oil recovery factor as a function of time, RF^{max} stands for the ultimate oil recovery factor, *t* represents imbibition time, and *a* is the parameter that must be determined.

The Aronofsky correlation has three assumptions: (1) oil recovery is a time-continuous function; (2) infinite oil recovery has a specified value; (3) all of the properties that affect the exit rate of oil from matrix block and ultimate oil recovery are almost constant during the mechanism (Aronofsky et al., 1958). The rate of oil that is produced from a matrix block is shown by a transfer function (Kazemi et al., 1992). In some cases the Aronofsky correlation predicts lower and higher recovery factors at early and late times respectively (Standnes, 2010b). This model is simple because only one parameter must be tuned. More complex expressions have been presented to better fit experimental data (Civan, 1998; Ghaedi and Riazi, 2016; Gupta and Civan, 1994; Kazemi et al., 1992; Li and Horne, 2002, 2000; Ma et al., 1995; Zhou et al., 2002). As an instance, Ma et al. (1995) changed the shape of the Aronofsky model by defining the dimensionless imbibition time:

$$\frac{RF(t)}{RF^{\max}} = \left(1 - e^{-at_D}\right) \tag{2}$$

$$t_D = t\sqrt{\frac{k}{\varphi}}\frac{\sigma_{ow}}{\sqrt{\mu_o \mu_w}}\frac{1}{L_c^2} \tag{3}$$

where t_D is dimensionless time, *k* stands for absolute permeability, ϕ represents fractional porosity, σ_{ow} is surface tension between oil and water, μ_w and μ_o represent the viscosity of water and oil respectively, and L_c is the characteristic length. Figure 1 shows a schematic of matrix blocks and their surrounding fractures. Based on this figure, L_c is defined as (Mirzaei-Paiaman et al., 2011):

$$L_c^2 = \frac{V_i}{\sum_{i=1}^{n}\frac{A_i}{L_i}} \tag{4}$$

where V_b is the bulk volume of the matrix block, A_i stands for the area of i^{th} imbibition surface, and L_i is the distance from i^{th} imbibition surface to the no-flow boundary.

This correlation is useful for any geometry and fluid of the matrix block. Equation (2) was fitted to imbibition experimental data on strongly water-wet rock samples (Ma et al. 1995), and the fitting parameter (a) was approximately equal to 0.05.

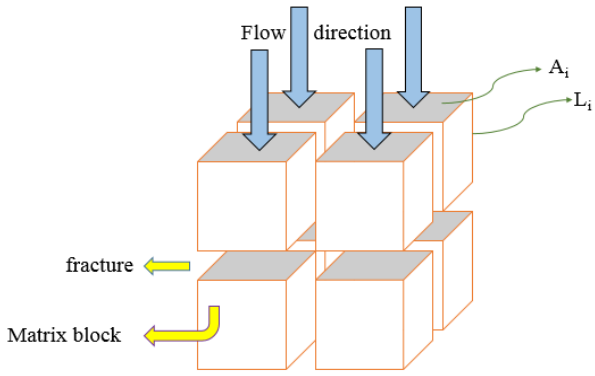

Figure 1

A schematic of matrix block and fractures.

The second model proposed to describe the imbibition process is Lambert W function (Standnes, 2010b). As mentioned above, the Aronofsky model underestimates the recovery factor at an early time and overestimates it at a late time. Fries and Dreyer (2008) proposed this model to modify the prediction of the oil recovery factor. They considered flow in the capillary tubes and solved Washburn equation for vertical flow by considering gravity force proportional to height; finally, they found a solution for the Washburn equation (Washburn 1921; Standnes, 2010b):

$$h(t) = \frac{m}{n}\left[1 + W\left(e^{-1-\frac{n^2}{m}}\right)\right] \tag{5}$$

$$m = \frac{2\sigma_{ow}\cos\theta}{\varphi\mu_w}\frac{k}{r} \tag{6}$$

$$n = \frac{\rho_w g k}{\varphi\mu_W} \tag{7}$$

where r is tube radius, ρ_w stands for water phase density, θ represents contact angle, and g is the acceleration due to gravity.

If Equation (5) is divided into the length of the capillary tube (L), term $\frac{m}{nL}$ (e.g. saturation) is known as the capillary rise to gravity head. Then, the equation is given by:

$$\frac{RF(t)}{RF^{\max}} = 1 + W\left(e^{-1-at}\right) \tag{8}$$

$$a = \frac{n^2}{m} \tag{9}$$

Thus, Lambert W function is a one-parameter model, which is more reliable due to considering the gravity force. The reason is that the gravity force in the reservoir is important at a later time of the depletion of the matrix block due to decreasing the capillary force. This model is suitable for water-wet cases (Standnes, 2010b).

The Lambert W function is a function with a domain of $[-e^{-1}, +\infty)$ and a range of $[-1, +\infty)$ and is expressed by:

$$x = W(x)e^{W(x)} \tag{10}$$

There is an equivalent for Lambert W function in domain $[-e^{-1}, 0]$ as given in Equation (11). In this domain, the maximum relative error is about 0.1%.

$$W(x) \approx -1 + \frac{\sqrt{2+2ex}}{1 + \frac{4.13501\sqrt{2+2ex}}{12.7036 + \sqrt{2+2ex}}} \tag{11}$$

where *e* represents the Euler number and is equal to 2.718282.

Dose-response models are utilized to demonstrate the degree of the response of an organism in terms of exposure to a stimulus such as chemical doses after a specific exposure time (Demidenko et al., 2017; Peleg et al., 1997; Sparling, 2016; Toth et al., 2013). The curve of dose-response behavior is very similar to the imbibition recovery curve. Several two-parameter models have been used in dose-response modeling including Logit, beta-Poisson, and Weibull. Hence, in this study, a complete analysis is conducted on the suitability of using these two-parameter dose-response models for describing the imbibition process. The performance of the available single-parameter models is also evaluated. The suitability of the models is examined based on a data bank of recovery curves collected for oil and gas reservoirs at different reservoir parameters. The rest of the paper is organized as follows. First, two-parameter dose-response models are explained. Then, the data bank of the recovery curves is presented. Finally, the performance of the different models in describing the recovery curves of the collected data is examined.

2. Two-parameter dose-response models

The imbibition recovery curve scaled with respect to recoverable oil in place has the following properties:

- At time zero, it equals zero, meaning that no oil is produced at the initial time.
- It is an increasing function with time, which means that a higher recovery factor is attained as time passes.
- After passing a considerable time, let say when *t* approaches infinity, this function has a value equal to one. This means that when *t* becomes indefinitely high, all of the producible oil is produced.

The properties mentioned for the scaled imbibition recovery curve are similar to those expected for dose-response modeling functions. Also, the shape of the resultant curves in both processes is similar. A typical dose-response model is shown in Figure 2. Therefore, the main purpose of this work is to answer the question whether these dose-response models can be utilized to predict oil/gas recovery during imbibition phenomena.

The probability of a definite response from exposure to a certain pathogen in terms of the dose is illustrated by the dose-response model (Demidenko et al., 2017; Sparling, 2016). Among the available sigmoid-shaped dose-response relationship, Logit, beta-Poisson, and Weibull models were used in this work for describing the imbibition recovery curves. In the following, these popular models are presented.

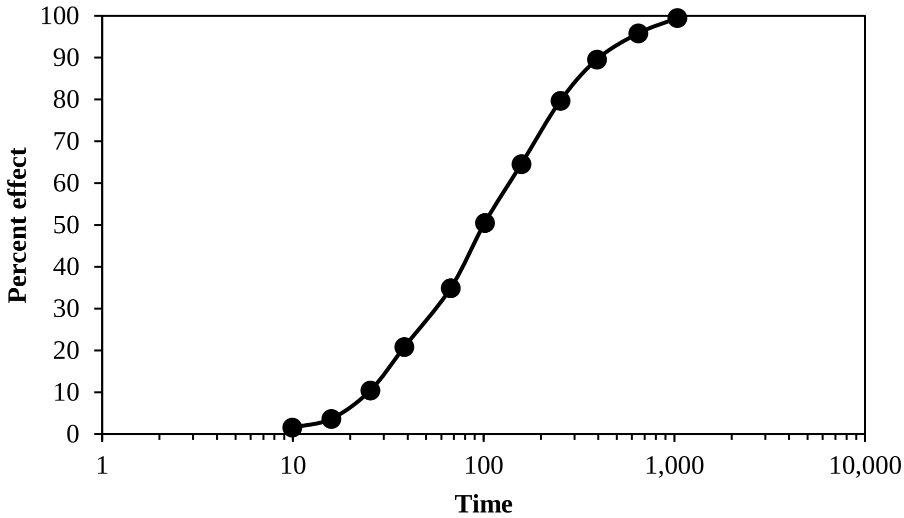

Figure 2

A typical dose-response model (Paul, 2015).

2.1. Weibull Model

From the probability theory point of view, the Weibull distribution is a continuous probability distribution. The cumulative distribution function of the Weibull distribution expressed in Equation (12) is one of the popular dose-response models (Carlborg, 1981):

$$\frac{RF(t)}{RF^{\max}} = 1 - e^{-at^{b}} \tag{12}$$

where a and b are the fitting parameters. This model has the three necessary properties mentioned for the imbibition recovery curve scaled with respect to recoverable oil in place.

2.2. Beta-Poisson model

In probability theory, the beta-Poisson distribution is a discrete probability distribution. The cumulative distribution function for the beta-Poisson model define as Equation (13) is another well-known dose-response model (Xie et al., 2017):

$$\frac{RF(t)}{RF^{\max}} = 1 - \left(1 + \frac{t}{a}\right)^{-b} \tag{13}$$

This model also has the three necessary properties mentioned for the imbibition recovery curve scaled with respect to recoverable oil in place.

2.3. Logit model

Based on the probability theory, a Logit-normal distribution has a normal distribution, naturally. This distribution is also recognized as the logistic normal distribution. The cumulative distribution function for the Logit dose-response model can be mentioned as one of the oldest models used for different purposes. This model is defined as (Demidenko et al., 2017):

$$\frac{RF(t)}{RF^{\max}} = \frac{1}{1+e^{a-b\ln(t)}} \tag{14}$$

It should be mentioned that the three necessary properties mentioned for the imbibition recovery curve scaled with respect to recoverable oil in place exist in this model, too.

3. Collected data bank

A data bank of the available experimental and numerical simulation imbibition recovery data with different rock and fluid properties were gathered from previous works (Fischer et al., 2008; Ghasemi, 2018; Standnes, 2010a; Xie and Morrow, 2001; Zhou et al., 2002). The collected data belong to both strongly water-wet and weakly water-wet rocks in oil and gas reservoirs. Table 1 tabulates properties, including permeability, porosity, oil viscosity, and water viscosity of the experimental and numerical data utilized for investigating the suitability of imbibition correlations. Figures 3–7 represent the imbibition recovery curves from Xie and Morrow (2001), Ghasemi.F (2018), Standnes (2010b), Fischer et al. (2008), and Zhou et al. (2002) respectively. As can be seen from the presented data, they cover a considerable range of reservoir rock and fluid properties. It should be highlighted that the recovery curves of Xie and Morrow (2001) result from weakly water-wet rocks, and those of Standnes (2010b) originate from strongly water-wet rocks.

Table 1

Properties of experimental and numerical data used for analyzing imbibition correlations.

Test No.	Source of data	K (mD)	ϕ (%)	μ_o (cP)	μ_w (cP)	Matrix phase
Test 1	Xie and Morrow (2001)	450	20.47	18.5	1	Oil
Test 2	Xie and Morrow (2001)	450	19.91	18.5	1	Oil
Test 3	Xie and Morrow (2001)	450	19.97	18.5	1	Oil
Test 4	Xie and Morrow (2001)	450	20.24	18.5	1	Oil
Test 5	Xie and Morrow (2001)	450	20.8	18.5	1	Oil
Test 6	Xie and Morrow (2001)	450	20.14	18.5	1	Oil
Test 7	Xie and Morrow (2001)	450	20.28	18.5	1	Oil
Test 8	Xie and Morrow (2001)	450	20.39	18.5	1	Oil
Test 9	Xie and Morrow (2001)	450	18.84	18.5	1	Oil
Test 10	Xie and Morrow (2001)	450	20.27	18.5	1	Oil
Test 11	Ghasemi.F (2018)	10	20	0.3985	0.224	Gas
Test 12	Ghasemi.F (2018)	10	30	0.3985	0.224	Gas
Test 13	Ghasemi.F (2018)	100	20	0.075	0.224	Gas
Test 14	Ghasemi.F (2018)	100	30	0.075	0.224	Gas
Test 15	Ghasemi.F (2018)	100	20	0.3985	0.224	Oil
Test 16	Ghasemi.F (2018)	100	30	0.3985	0.224	Oil
Test 17	Ghasemi.F (2018)	10	20	0.075	0.224	Oil
Test 18	Ghasemi.F (2018)	10	30	0.075	0.224	Oil
Test 19	Standnes (2010b)	10	25	1	2	Oil
Test 20	Standnes (2010a)	10	25	1	2	Oil
Test 21	Standnes (2010a)	1000	25	1	2	Oil

Test No.	Source of data	*K* (mD)	*ϕ* (%)	μ_o (cP)	μ_w (cP)	Matrix phase
Test 22	Standnes (2010a)	100	25	1	2	Oil
Test 23	Standnes (2010a)	10	25	1	2	Oil
Test 24	Fischer et al. (2008)	62.3	16.9	3.9	1	Oil
Test 25	Fischer et al. (2008)	62.7	16.8	3.9	4.1	Oil
Test 26	Fischer et al. (2008)	112.3	18.6	3.9	27.8	Oil
Test 27	Fischer et al. (2008)	148.4	19.2	3.9	97.7	Oil
Test 28	Fischer et al. (2008)	136.7	19.1	3.9	1	Oil
Test 29	Fischer et al. (2008)	143.9	19.1	63.3	4.1	Oil
Test 30	Zhou et al. (2002)	510.8	21.8	37.82	0.967	Oil
Test 31	Zhou et al. (2002)	498.5	21.9	37.82	0.967	Oil
Test 32	Zhou et al. (2002)	519.8	22.2	37.82	0.967	Oil
Test 33	Zhou et al. (2002)	521.7	22.4	37.82	0.967	Oil
Test 34	Zhou et al. (2002)	505.5	21.5	37.82	0.967	Oil
Test 35	Zhou et al. (2002)	501.6	21.8	37.82	0.967	Oil

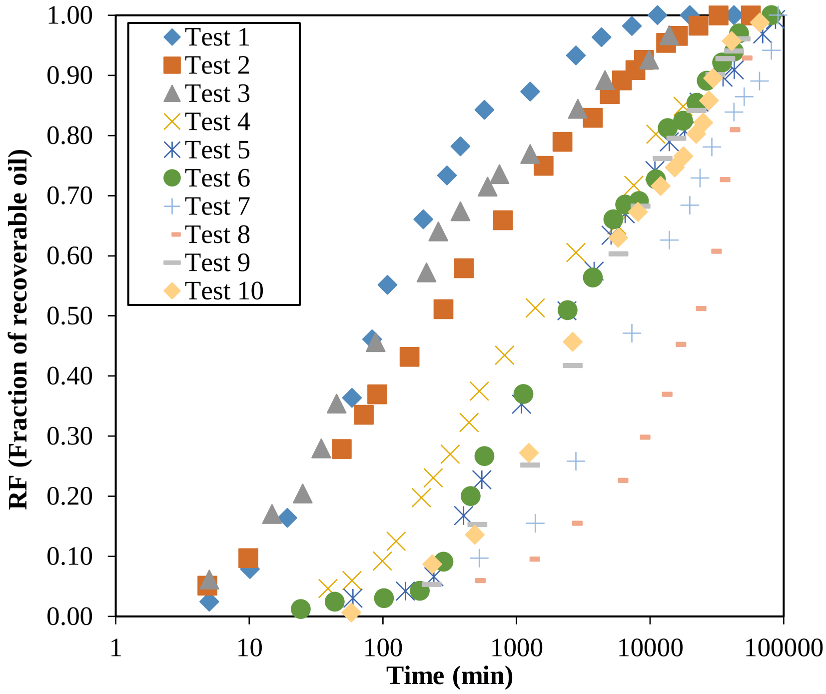

Figure 3

Recovery curves of weakly water-wet rocks from Xie and Morrow (2001).

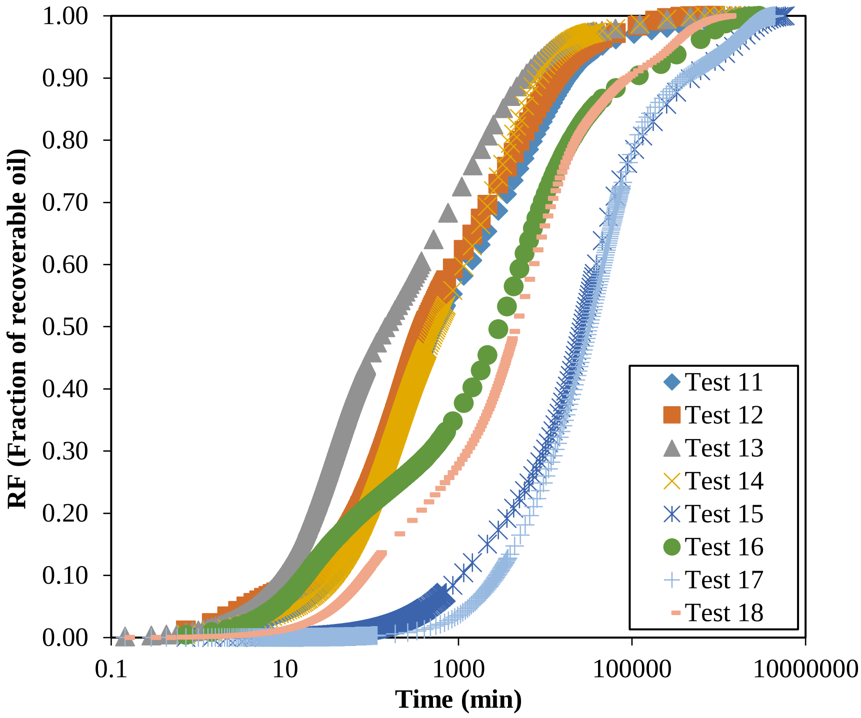

Figure 4

Recovery curves resulted from the simulation of a single matrix block filled with oil and gas (Ghasemi.F 2018).

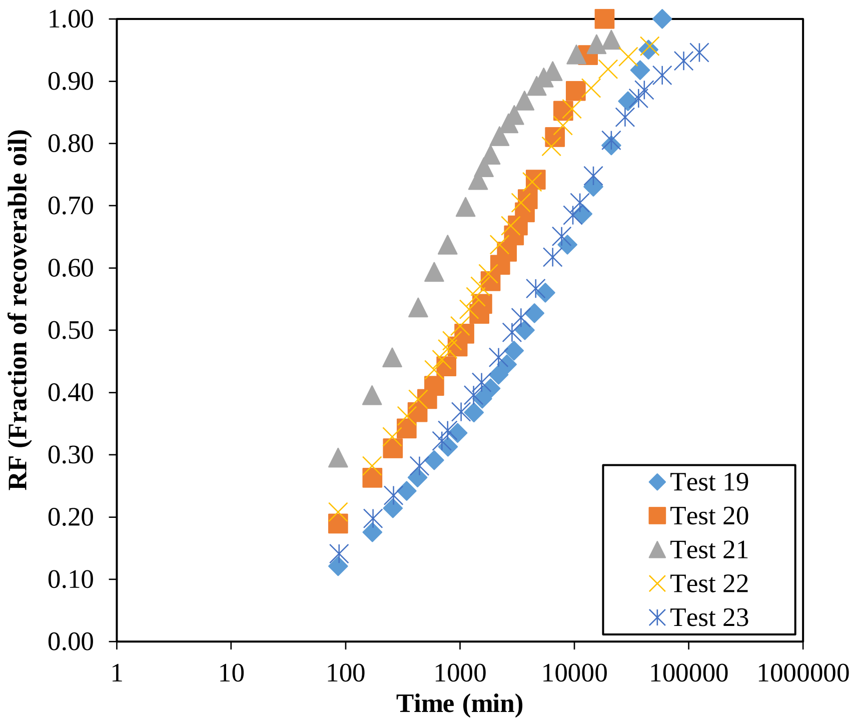

Figure 5

Recovery curves of strongly water-wet rocks from Standnes (2010a).

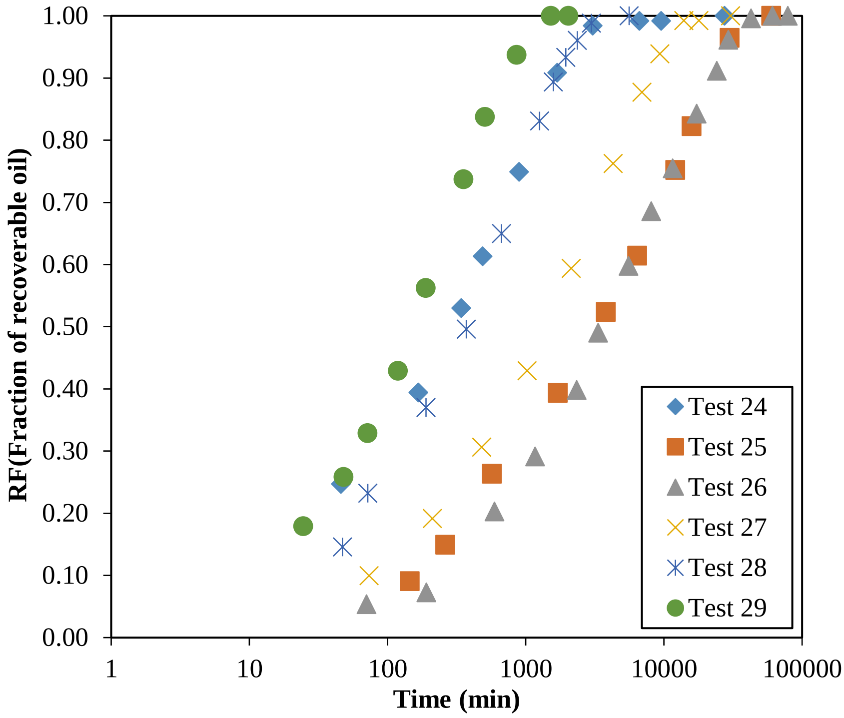

Figure 6

Recovery curve of the cases presented by Fischer et al. (2008).

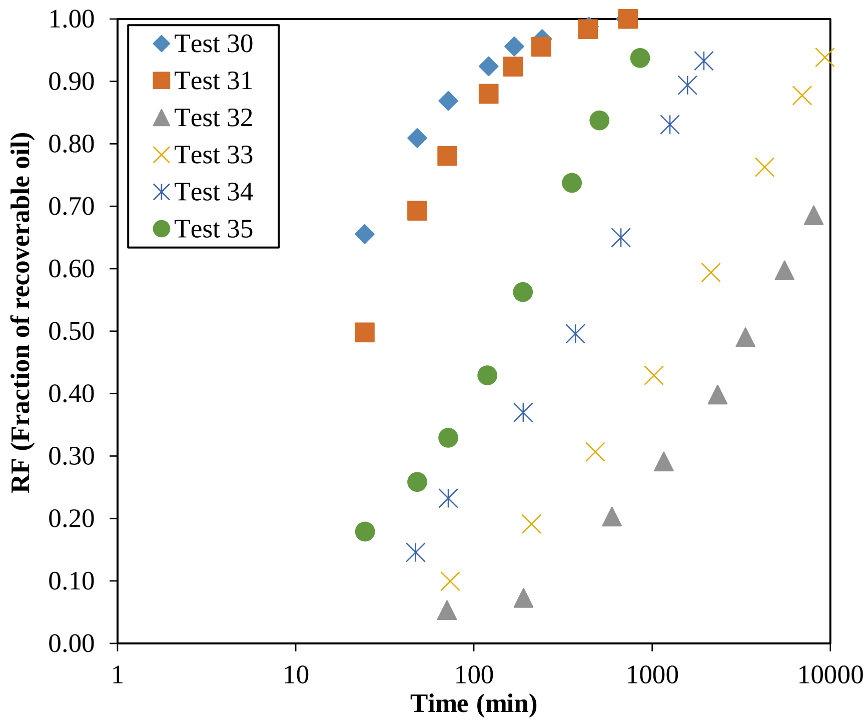

Figure 7

Recovery curves of the cases presented by Zhou et al. (2002).

4. Results and discussions

Table 2 lists the regression results of the single- and two-parameter models fitted to the data bank of imbibition recovery curves. In the majority of the cases, based on the calculated R^2, one can conclude that fairly good results are obtained for the single-parameter models. Lambert W function model gains the superiority over the other single-parameter model, i.e. the Aronofsky model. In some cases, such as tests 20–23, which present the recovery curves of strongly water-wet rocks, the results of the Aronofsky model are not acceptable. Lambert W function also fails in some cases such as tests 16 and 30.

Table 2

Results of the fitting single- and two-parameter models to the imbibition data.

Test No	Aronofsky		Lambert W function		Weibull			Beta-Poisson			Logit		
	A	R^2	*a*	R^2	*a*	*b*	R^2	*a*	*b*	R^2	*a*	*b*	R^2
1	0.00578	0.956	0.00201	0.947	0.03135	0.65589	0.985	75.8	0.83023	0.998	4.40097	0.95361	0.997
2	0.00256	0.832	0.00056	0.864	0.06861	0.40597	0.993	61.8	0.46578	0.988	3.55948	0.64767	0.995
3	0.00379	0.764	0.00120	0.852	0.07831	0.42735	0.971	28.5	0.41799	0.995	3.21475	0.64990	0.991
4	0.00050	0.699	0.00011	0.903	0.01864	0.48230	0.973	271.1	0.39703	0.993	4.94730	0.67670	0.986
5	0.00020	0.908	0.00006	0.951	0.00736	0.56407	0.985	1381.8	0.67035	0.994	6.66956	0.84539	0.994
6	0.00020	0.908	0.00006	0.951	0.00736	0.56407	0.985	1381.8	0.67035	0.994	6.66956	0.84539	0.994
7	0.00006	0.946	0.00002	0.986	0.00157	0.66778	0.995	11299.8	1.23726	0.990	9.28818	1.03206	0.988
8	0.00004	0.974	0.00002	0.928	0.00006	0.95784	0.974	43539628.3	1673.04938	0.974	12.66341	1.30146	0.952
9	0.00014	0.941	0.00005	0.977	0.00312	0.64694	0.995	3980.1	1.04245	0.997	8.18123	0.99553	0.997
10	0.00012	0.898	0.00005	0.964	0.00419	0.60842	0.980	2601.1	0.79566	0.987	7.51367	0.91327	0.986
11	0.00174	0.915	0.00049	0.929	0.02027	0.56279	0.971	136.5	0.45020	0.998	4.61363	0.73827	0.992
12	0.00201	0.892	0.00061	0.943	0.02604	0.54215	0.982	110.2	0.45424	0.996	4.33474	0.71924	0.996
13	0.00777	0.917	0.00202	0.947	0.04412	0.56815	0.977	29.5	0.42576	0.996	3.52329	0.72964	0.991
14	0.00173	0.926	0.00048	0.960	0.01921	0.57194	0.989	177.9	0.53175	0.996	4.80560	0.77293	0.997
15	0.00003	0.976	0.00001	0.984	0.00093	0.64760	0.998	8627.6	0.52641	0.993	7.74410	0.76202	0.999
16	0.00104	0.643	0.00021	0.790	0.04085	0.36396	0.993	96.6	0.24691	0.928	3.72512	0.49000	0.981
17	0.00002	0.995	0.00001	0.985	0.00041	0.71538	0.998	23348.2	0.84043	1.000	9.66204	0.93632	1.000
18	0.00019	0.952	0.00004	0.975	0.01193	0.48488	0.998	386.1	0.38241	0.987	5.15271	0.64209	0.997
19	0.00020	0.702	0.00006	0.919	0.01623	0.46357	0.990	705.5	0.44968	0.937	5.25638	0.65967	0.966
20	0.00048	0.643	0.00017	0.922	0.02321	0.48431	0.990	332.2	0.51081	0.945	4.89766	0.70764	0.967
21	0.00126	0.639	0.00045	0.809	0.05012	0.45115	0.997	144.7	0.59488	0.987	4.32558	0.74733	0.995
22	0.00062	0.649	0.00020	0.844	0.03597	0.43164	0.999	246.3	0.47041	0.974	4.50912	0.66513	0.992
23	0.00020	0.656	0.00006	0.852	0.02578	0.41425	0.999	478.2	0.40473	0.968	4.81240	0.61601	0.990
24	0.00210	0.931	0.00080	0.973	0.02183	0.61647	0.994	327.7	1.16265	0.973	5.27291	0.95381	0.972
25	0.00017	0.900	0.00006	0.987	0.00656	0.57694	0.994	1789.9	0.75107	0.970	6.56552	0.82916	0.978
26	0.00017	0.956	0.00006	0.994	0.00307	0.66121	0.998	4480.8	1.23489	0.989	8.11869	1.00490	0.987
27	0.00045	0.959	0.00016	0.994	0.00538	0.67394	0.998	1933.5	1.38959	0.986	7.29471	1.03234	0.983
28	0.00177	0.967	0.00070	0.994	0.00931	0.73820	0.996	770.9	1.94559	0.986	6.43346	1.12755	0.978
29	0.00446	0.971	0.00165	0.992	0.01511	0.76615	0.999	335.5	1.97440	0.992	5.60751	1.14512	0.984
30	0.03717	0.841	0.01556	0.673	0.18877	0.55064	0.994	20.9	1.37951	0.999	3.19216	1.19900	0.998
31	0.02359	0.934	0.00980	0.920	0.08144	0.68114	0.997	52.0	1.79038	0.999	4.12539	1.28125	0.997
32	0.00019	0.908	0.00007	0.993	0.00335	0.65115	0.998	1483.7	0.58856	0.993	6.78756	0.83365	0.997

33	0.00045	0.932	0.00017	0.999	0.00589	0.66126	0.998	1315.0	1.05722	0.985	6.76400	0.95490	0.985
34	0.00178	0.948	0.00070	0.995	0.01013	0.72344	0.995	537.2	1.47431	0.983	5.98844	1.04690	0.979
35	0.00446	0.961	0.00167	0.990	0.01545	0.76153	0.999	282.9	1.72130	0.991	5.43304	1.10786	0.985
Average	0.0031	0.873	0.0012	0.934	0.0258	0.5927	0.991	1245967.4	48.6381	0.985	5.8873	0.8732	0.988

Two-parameter models, namely Weibull, beta-Poisson, and Logit, excellently match the experimental/numerical data for all the cases of the data bank, and the Weibull model has the best performance among the two-parameter models. It should be noted that since usually considerable data points exist for imbibition processes, it makes sense to fit two-parameter models to these data. Due to the complexity of fractured reservoir production, single-parameter correlations sometimes fail to well or fairly well match the imbibition data.

For a better illustration of the quality of the fitting resulted from the single- and two-parameter models, the behavior of these models in two tests, i.e. tests 10 and 26, is delineated in Figures 8 and 9 respectively. It is clear that the Aronofsky model underestimates the recovery factor in short time frames and overestimates it at longer times, which has been pointed out by Standnes (2010b) as well. Lambert W function presents a very good match in both tests. The problem with this model is the prediction at long times, and it leads to a value greater than one when the time approaches infinity. Therefore, one must be careful when using this function to predict the recovery factor at long times. Actually, this function does not have the ability to approach one when a considerable time is passed.

The beta-Poisson model presents a behavior which is opposite to that of the Aronofsky model. It overestimates the recovery factor in short time frames and underestimates it at longer times. Both the Weibull and Logit models well match the imbibition data in the presented tests, and especially the Weibull model makes a more acceptable prediction.

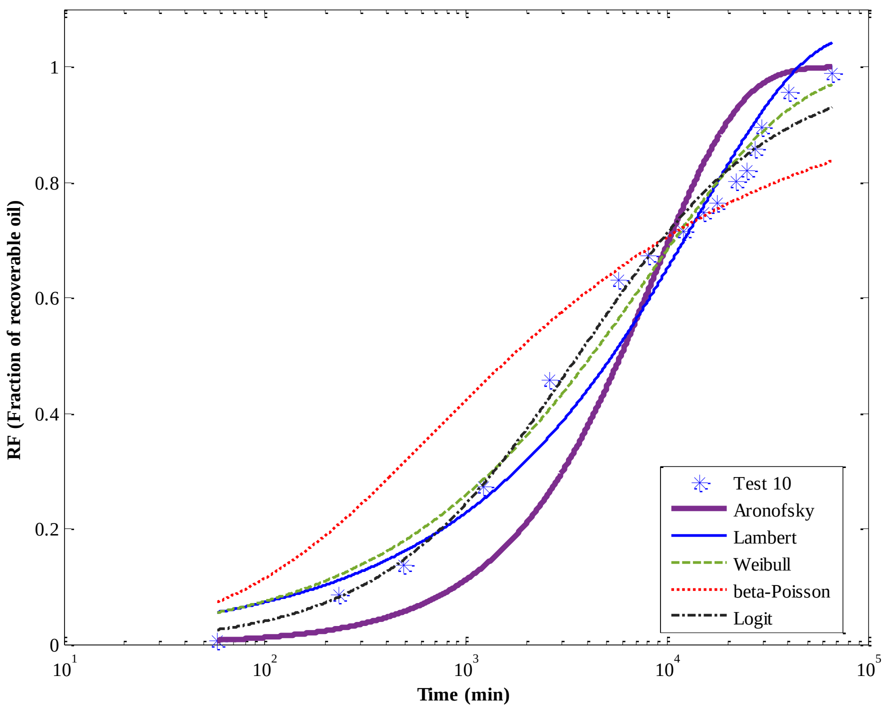

Figure 8

Comparison of oil recovery factor prediction by different models for Test 10.

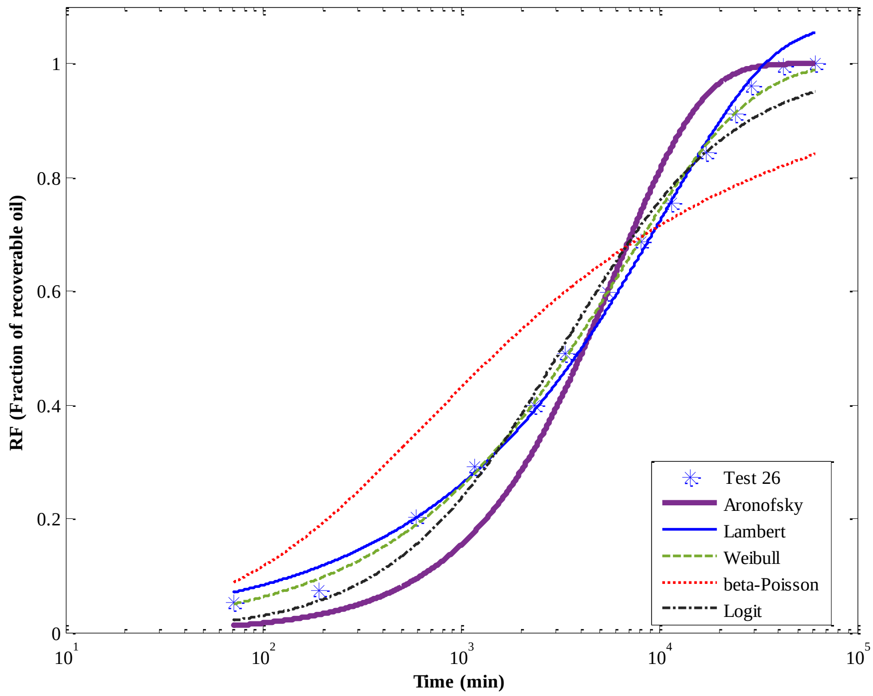

Figure 9

Comparison of oil recovery factor prediction by different models for Test 26.

5. Conclusions

In this work, a comprehensive analysis was conducted on the suitability of single- and two-parameter models for describing imbibition recovery curves. A data bank from available imbibition recovery curves with different reservoir properties was built. Then, single-parameter models, including the Aronofsky and Lambert W function, and the two-parameter models from dose-response modeling such as beta-Poisson, Logit, and Weibull were fitted to the imbibition data. The results reveal that the Aronofsky model underestimates the recovery factor in short time frames and overestimates it at longer times. Also, Lambert W function is not suitable for recovery prediction at longer times because it figures out values greater than one for the normalized recovery factor at long times. The beta-Poisson has the opposite behavior of the Aronofsky model, that is, it overestimates the recovery factor at short times and underestimates it at long times. Both the Weibull and Logit models well match the imbibition recovery curves. The performance of the Weibull model is more acceptable compared to the Logit model. Finally, it should be highlighted that the resultant average R^2 values for the Aronofsky, Lambert W function, Weibull, beta-Poisson, and Logit models are 0.873, 0.934, 0.991, 0.985 and 0.988 respectively.

Nomenclature

a	Fitting parameter
A_i	Area of i^{th} imbibition surface
B	Fitting parameter
G	Acceleration due to gravity
K	Absolute permeability
L_c	Characteristic length.
L_i	Distance from i^{th} imbibition surface to the no-flow boundary

r	Tube radius
RF	Oil recovery factor
RF^{max}	Ultimate oil recovery factor
t	Imbibition time
t_D	Dimensionless time
V_b	Bulk volume of the matrix block
ϕ	Porosity
μ_o	Oil viscosity
μ_w	Water viscosity
ρ_w	Water phase density
σ_{ow}	Surface tension between oil and water
θ	Contact angle

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