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A Least Squares Approach to Estimating the Average Reservoir Pressure

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Abstract

Least squares method (LSM) is an accurate and rapid method for solving some analytical and numerical problems. This method can be used to estimate the average reservoir pressure in well test analysis. In fact, it may be employed to estimate parameters such as permeability (k) and pore volume (Vp). Regarding this point, buildup, drawdown, late transient test data, modified Muskat method, interference test, and other methods are equivalent to a separable least squares problem. The main advantage of LSM in the well testing problems is that the results would be confident and no trial and error is required. Furthermore, the given method requires a short time. The fast rate of convergence and high accuracy of the LSM are demonstrated through two examples. The current study concerns a modified Muskat method. The results of LSM combined with the modified Muskat method are compared with the other iterative and qualitative methods. The preliminary numerical results with both simulated and field data suggest that the method be capable of producing smooth interpretable estimates of reservoir parameters from data.

Keywords: Least Squares Method, Average Reservoir Pressure, Modified Muskat Method, Rate of Convergence

1. Introduction

The purpose of well test analysis is to determine the geological and reservoir properties of hydrocarbon or water reservoirs by measuring wellbore pressure and production rate over time. There are several methods for the analysis of these measurements including buildup test, drawdown test, Miller-Dyes-Hutchinson (MDH) method, Matthews-Brons-Hazebroek (MBH) method, reservoir limit test, late transient test, fall-off test in injection wells, modified Muskat method, Russel method, interference test, and so on. The aim of buildup test analysis is determining permeability, average pressure ($\overline{p}$), false pressure (p^*), initial pressure (p_i), etc. (Earlougher, 1977; Verga et al., 2011; Chaudhry, 2004). The important application of MBH method is to determine average pressure. Russel method has been developed for using pressure buildup data at early times (Matthews and Russell, 1967). The other methods are also applicable to the system identification of reservoir and reservoir parameter evaluation (Kuchuk et al., 2010; Ahmed and Meehan, 2012). In 1937, Muskat performed a mathematical analysis on the basis of which he proposed that a plot of $\log\left(\overline{p}-p_{ws}\right)$ versus the closed-in time would give a straight line. The mathematical analysis was for the case of an incompressible flow, and thus it was not quantitatively applicable to actual reservoirs. However, the type of the plot suggested by Muskat was found applicable in the case of a compressible flow as will

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be discussed later as the modified Muskat method. Pollard used this type of plot for pressure buildup in fracture limestone reservoirs (Pollard, 1959). Other researchers employed it for pressure fall-off behavior in injection wells (Verga et al., 2011). These methods have a number of serious limitations which are discussed below.

First, plotting the pressure, production rate measurement data, or any functions of them versus time, cumulative production, or any functions of time and matching the obtained data points give qualitative and inaccurate results. Second, in most cases, these techniques require large computation time and are very memory-extensive.

In this paper, we define and discuss a powerful method for the analysis of these data to estimate $\overline{p}$, k, and V_p. LSM is a famous method now used in the numerical analyses. LSM is used in many cases whether for a given continuous function or some given data which will be considered in this work. In this approach, an interpolating function is used. The application of this method (LSM) is confident, since in applied mathematics the theory and applications of the LSM has been mentioned and proved (Li and Jia, 2001; Markovsky and Huffel, 2005).

The current work is organized in six sections; section 1 and section 2 present an introduction and a brief discussion about the LSM respectively. The problem statement is given in section 3. In section 4, the application of LSM in the analysis of reservoir pressure is discussed. The numerical simulations of the stated problem are provided in section 5. Finally, some concluding remarks close the paper in section 6.

2. The least squares method

Before any discussion about LSM, a useful definition is given as follows:

Definiton 1 (Boyd and Xu, 2009). The functions set $\{\psi_0,\psi_1,\ldots,\psi_n\}$, in which each function is continuous over an interval I, is a Chebyshev set on I (or satisfies the Haar condition) if for each choice of $a_0,a_1,\ldots,a_n$, which are not all zero, the function does not have more than n roots on I.

$$\Phi_a(x)=\sum_{r=0}^{n}a_r\psi_r(x) \tag{1}$$

The LSM is based on the idea that one wants to approximate a function ($f(x)$) on a finite set of grid points $\{x_0,x_1,\ldots,x_N\}$ by another function such as $\Phi_a(x)$ in the following sense. Let assume that $\{\psi_0,\psi_1,\ldots,\psi_n\}$, in which $n<N$, is a Chebyshev set on a closed interval of $[a,b]$ that contains all x_i's. Now, let evaluate the minimum of the following function for every real number of $a_0,a_1,\ldots,a_n$:

$$E(a_0,a_2,\ldots,a_n)=\sum_{i=0}^{N}\left(f(x_i)-\Phi_a(x_i)\right)^2 \tag{2}$$

This function is called the LSM index which is required to be minimized.

The necessary conditions for E to have a minimum is that in the minimum point the relation of $\frac{\partial E}{\partial a_r}=0,\ r=0,1,\ldots,n$ should hold. So

$$-2\sum_{i=0}^{N}\Big(\big(f(x_i)-\big(a_0\psi_0(x_i)+a_1\psi_1(x_i)+\ldots+a_n\psi_n(x_i)\big)\big)\psi_r(x_i)\Big)=0,\ r=0,1,\ldots n \tag{3}$$

which reduces to:

$$a_0\sum_{i=0}^{N}\left(\psi_0(x_i)\psi_r(x_i)\right)+a_1\sum_{i=0}^{N}\left(\psi_1(x_i)\psi_r(x_i)\right)+\ldots+a_n\sum_{i=0}^{N}\left(\psi_n(x_i)\psi_r(x_i)\right)=$$
$$\sum_{i=0}^{N}\left(f(x_i)\psi_r(x_i)\right),\ r=0,1,\ldots n \tag{4}$$

This n+1 systems of linear algebraic equations with n+1 unknowns $a_0,a_1,\ldots,a_n$ is called normal equations. It can be easily proved that the normal equations have unique solutions, if there is not any sets of real numbers like $b_0,b_1,\ldots,b_n$ (unless $b_0=b_1=\ldots=b_n$) such that

$$\sum_{r=0}^{n}b_r\psi_r(x_i)=0,\quad 0\le i\le N \tag{5}$$

Because $n<N$ and $\psi_r(x)$'s create a Chebyshev set, this condition is satisfied (Bjorck, 1996).

In this work, for the base function $\{\psi_0,\psi_1,\ldots,\psi_n\}$, we use the set of {1, x} which is clearly a Chebyshev set. Therefore, our approximated function is a straight line such as ax+b.

3. The problem statement

One solution of small and constant fluid compressibility for a radial flow is the bounded cylindrical reservoir. The flowing pressure at the wellbore (p_{wf}) for the case where r_e>>r_w is as follows (van Everdingen and Hurst, 1949)

$$p_{wf}=p_i-\frac{q\mu}{2\pi kh}\left(\frac{2t_{Dw}}{r_{eD}}+\ln(r_{eD})-\frac{3}{4}+2\sum_{n=1}^{\infty}\left(\frac{e^{\left(-\alpha_n^2t_{Dw}\right)}J_1^2(\alpha_nr_{eD})}{\alpha_n^2\left(J_1^2(\alpha_nr_{ED})-J_1^2(\alpha_n)\right)}\right)\right) \tag{6}$$

The modified Muskat method is used for estimating average reservoir pressure and pore volume drained by a well. The first assumption is that the well has been produced long enough to reach semi-steady state prior to shut-in stage; at this time, all exponential terms have died out. Next, the well is closed in for a time so that all but one of the exponential terms in Equation 7 die out. This occurs after the pressure buildup curve deviates from a straight line on a semi-log scale and starts to flatten out. It may be completed by superposing Equation 7 for these flowing and production times, and introducing the relation between $\overline{p}$ and p_i. On evaluating the Bessel functions for a large outer boundary, it would be found that:

$$\log\left(\overline{p}-p_{ws}\right)=\log\left(118.6\frac{q\mu B}{kh}\right)-0.00168\frac{k\,\Delta t}{\phi\mu cr_e^2} \tag{7}$$

The units in this equation are the practical oilfield units. Also, approximations used in developing this equation are valid in the shut-in time range too:

$$\frac{250\phi\mu c_t r_e^2}{k}\le\Delta t\le\frac{750\phi\mu c_t r_e^2}{k} \tag{8}$$

If there are not any estimates of k and r_e, the data corresponding to times long enough can also be considered. It can be seen from Equation 8 that a plot of $\log\left(\overline{p}-p_{ws}\right)$ versus Δt should be linear with a slope of $a=0.00168\dfrac{k}{\varphi\mu cr_e^2}$ and an intercept of $b=118.6\dfrac{q\mu B}{kh}$ on the semi-log scale.

Frequently in this type of analysis, the average pressure $\overline{p}$ is not known. This may be obtained from a trial-and-error plot that will be improved by a new method as a separable least squares method in the next section.

4. Application of LSM in reservoir pressure analysis

The modified Muskat method can be regarded as a least squares problem in which the index that should be minimized is as follows:

$$E\left(\overline{p},a,b\right)=\sum_{i=1}^{N}\left[\log\left(\overline{p}-p_{ws}\right)-\left(a\Delta t_i+b\right)\right]^2 \tag{9}$$

where, the parameters $\overline{p}$, a, and b should be derived. The necessary conditions for minimizing E are:

$$\frac{\partial E}{\partial a}=0,\quad \frac{\partial E}{\partial b}=0,\quad \frac{\partial E}{\partial \overline{p}}=0 \tag{10}$$

Therefore, these conditions yield:

$$\sum_{i=1}^{N}\left(\Delta t_i\left(\log\left(\overline{p}-p_{wsi}\right)-a\Delta t_i-b\right)\right)=0 \tag{11}$$

$$\sum_{i=1}^{N}\left(\log\left(\overline{p}-p_{wsi}\right)-a\Delta t_i-b\right)=0 \tag{12}$$

$$\sum_{i=1}^{N}\left(\frac{1}{\left(\overline{p}-p_{wsi}\right)}\left(\log\left(\overline{p}-p_{wsi}\right)-a\Delta t_i-b\right)\right)=0 \tag{13}$$

By setting:

$$\alpha=\sum_{i=1}^{N}\left(\Delta t_i\right)^2 \tag{14}$$

$$\beta=\sum_{i=1}^{N}\left(\Delta t_i\right) \tag{15}$$

From Equations 12, 13, 15, and 16, one can easily obtain:

$$\alpha a+\beta b=\sum_{i=1}^{N}\left(\Delta t_i\log\left(\overline{p}-p_{wsi}\right)\right)=\log\prod_{i=1}^{N}\left(\overline{p}-p_{wsi}\right)^{\Delta t_i}=f\left(\overline{p}\right) \tag{16}$$

$$\beta a+Nb=\sum_{i=1}^{N}\left(\log\left(\overline{p}-p_{wsi}\right)\right)=\log\prod_{i=1}^{N}\left(\overline{p}-p_{wsi}\right)=g\left(\overline{p}\right) \tag{17}$$

By solving the above system of linear algebraic equations, the regression parameters a and b are obtained as:

$$a=\frac{Nf\left(\overline{p}\right)-\beta g\left(\overline{p}\right)}{N\alpha-\beta^2}=a\left(\overline{p}\right) \tag{18}$$

$$b=\frac{g\left(\overline{p}\right)-\beta a\left(\overline{p}\right)}{N}=b\left(\overline{p}\right) \tag{19}$$

To determine the third parameter, substituting Equations 19 and 20 in Equation 14 yields:

$$h\left(\overline{p}\right)=\sum_{i=1}^{N}\left(\frac{1}{\left(\overline{p}-p_{wsi}\right)}\left(\log\left(\overline{p}-p_{wsi}\right)-a\left(\overline{p}\right)\Delta t_i-b\left(\overline{p}\right)\right)\right)=0 \tag{20}$$

from which, the third parameter ($\overline{p}$) can be found via the following algorithm:

$$\overline{p}_{j+1}=\overline{p}_j-\frac{h\left(\overline{p}_j\right)}{h'\left(\overline{p}_j\right)} \tag{21}$$

where,

$$h'(\overline{p})=\sum_{i=1}^{N}\left(\frac{-1}{\left(\overline{p}-p_{wsi}\right)^{2}}(\log\left(\overline{p}-p_{wsi}\right)-a\left(\overline{p}\right)\Delta t_i-b\left(\overline{p}\right)+\frac{1}{\left(\overline{p}-p_{wsi}\right)}\left(\frac{1}{\ln(10)\left(\overline{p}-p_{wsi}\right)}-a'\left(\overline{p}\right)\Delta t_i-b'\left(\overline{p}\right)\right)\right) \quad (22)$$

$$a'\left(\overline{p}\right)=\frac{Nf'\left(\overline{p}\right)-\beta g'\left(\overline{p}\right)}{N\alpha-\beta^2} \quad (23)$$

and

$$b'\left(\overline{p}\right)=\frac{g'\left(\overline{p}\right)-\beta a'\left(\overline{p}\right)}{N} \quad (24)$$

in which,

$$f'\left(\overline{p}\right)=\sum_{i=1}^{N}\frac{\Delta t_i}{\ln(10)\left(\overline{p}-p_{wsi}\right)} \quad (25)$$

$$g'\left(\overline{p}\right)=\sum_{i=1}^{N}\frac{1}{\ln(10)\left(\overline{p}-p_{wsi}\right)} \quad (26)$$

5. Numerical examples

To illustrate the generality and accuracy of the proposed LSM-based technique, the modified Muskat method is considered under various assumptions. These problems have been considered by other researchers as well (Larson, 1963; Al-Attar and Abdul-Majeed, 1988). The results are also compared with the actual values or those of the same problems solved by others (Crump and Hite, 2008).

Example 1: The data required for verifying this method are tabulated in Tables 1 and 2 (Lee, 1982). The data in Tables 1 and 2 were obtained in a pressure buildup test on an oil well producing above the bubble point. The other properties of the well and reservoir are given in Table 3.

Table 1

Oil well pressure buildup test data

Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)
0	3534	2	4250	20	4379
0.15	3680	4	4320	24	4384
0.2	3723	6	4340	30	4393
0.3	3800	7	4344	40	4398
0.4	3866	8	4350	50	4402
0.5	3920	12	4364	60	4405
1	4103	16	4373	72	4407

Table 2

Data point that have been used

Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)
30	4393	60	4405
40	4398	72	4407
50	4402		

Table 3
Reservoir and fluid properties for Example 1

Property	Value	Property	Value
t_p (hour)	13,630	B_0 (RB/STB)	1.136
h (ft)	69	c_t (psi^{-1})	1.7×10^{-5}
r_w (ft)	0.198	μ_0 (cp)	0.8
r_e (ft)	1320	ϕ	0.039
q_0 (STB/bbl)	250		

In this case, the estimates of k and r_e are fortunately available; thus, it can eliminate the data outside the time range of interest. k and r_e are not often known. Therefore,

$$\frac{250\phi\mu_0 c_t r_e^2}{k}=\frac{(250)(1320)^2}{1.442\times10^7}=30.2$$

and

$$\frac{750\phi\mu_0 c_t r_e^2}{k}=90.6$$

Thus, it may be approximated from $\Delta t=30$ hours until recording pressures is stopped, i.e. $\Delta t=72$ hours.

The result of the qualitative method was 4412 psia (Larson, 1963). But, the results of LSM are 4411.4764 psia. Figure 1 shows the analysis results. The best straight line that can be obtained by LSM is given in Figure 1. Now from a and b, one can calculate kh and V_p that are shown in Table 4 (modified Muskat method).

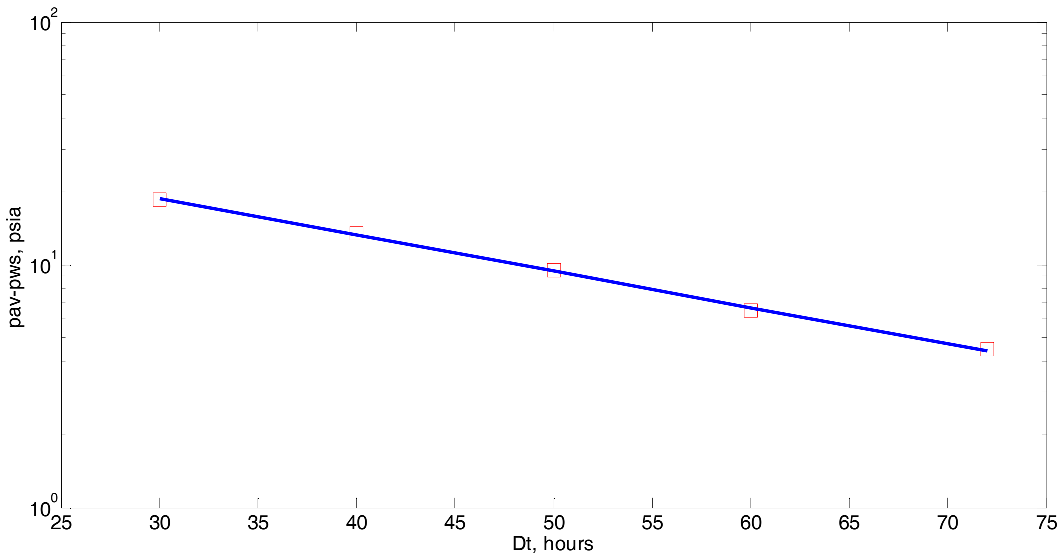

Figure 1
Data point and straight line obtained by least squares method on semi-log scale

The validation of this method for $\overline{p}$=4411.4764 psia is given in Table 5 and Figure 1; it can be compared with the previous method (trial and error) as displayed in Table 6 and Figure 2 ($\overline{p}$=4412). The percentage error can be calculated by the following relation:

$$Percentage\ error=\frac{Our\ result-Existing\ \ Solution}{Existing\ \ Solution}\times100\%$$

The summation of difference squared between column 2 and column 3 of Table 5 (error) is equal to:

$$\sum_{i=1}^{5}\left[\left(\bar{p}-p_{wsi}\right)-a\Delta t_i-b\right]^2=2.8425\times10^{-4}$$

Table 4

Calculated parameters

Parameter	Value
a (psia/hour)	-0.0149
b (psia)	52.1738
kh (md-ft)	516.4646
V_p(bbl)	3.4494×10^{8}

However, this value calculated for Table 6 is equal to 3.1784×10^{-4}. Both values are acceptable, but the first value is closer to zero and more accurate. However, the trial and error method usually needs a lot of time to give an accurate result, which is not economically acceptable. It should be noted that the third column in Tables 5 and 6 is the values of straight line that may be found in a semi-log plot based on the modified Muskat theory.

Table 5

Estimation of average pressure accuracy, a and b values, using our method (LSM)

Δt	$\log\left(\bar{p}-p_{ws}\right)$	$a\Delta t+\log(b)=-0.0149\Delta t+1.7175$	Percentage error
30	1.2666	1.2705	0.3079
40	1.1296	1.1215	-0.7171
50	0.9766	0.9725	-0.4198
60	0.8113	0.8235	1.5038
72	0.6509	0.6447	-0.9525

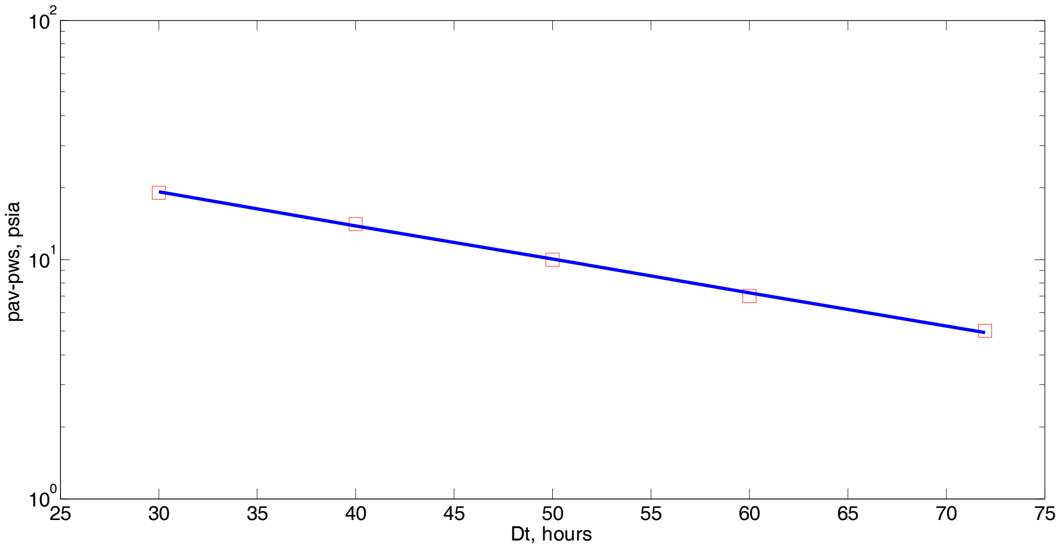

Figure 2

Data point and straight line obtained by trial and error method on semi-log scale

Table 6
Estimation of average pressure accuracy, a and b values, using trial and error method

Δt	$\log(\bar{p}-p_{ws})$	$a\Delta t+\log(b)=-0.0140\Delta t+1.7008$	Percent error
30	1.2788	1.2808	0.1564
40	1.1461	1.1408	-0.4624
50	1.0000	1.0008	0.0800
60	0.8451	0.8608	1.8578
72	0.6990	0.6928	-0.8870

This best straight line indicates the most accurate $\bar{p}$, a, and b. Then, from the given formula (modified Muskat method) for kh and V_p, the accuracy of a and b guarantees the accuracy of kh and V_p. However, a comparison between the calculated and real values can prove this claim, which is given in Table 7. The computation time for our method (LSM) is 0.002063 seconds. However, the time of computation of trial and error method is variable and usually too much.

Table 7
Comparison of parameters using conventional (trial and error) and our calculated (LSM) methods

	kh	V_p
Conventional	527.7374	3.7770×10^{8}
Calculated	516.4646	3.4494×10^{8}
Percentage error	-2.1361	-8.6736

Example 2: The other example is a synthetic case for which the reservoir and fluid properties are given in Table 8. The time range of interest is estimated as follows:

$$\frac{250\varphi\mu_o c_t r_e^2}{k}=\frac{250(4000)^2}{6.6667\times10^7}=60$$

and

$$\frac{750\varphi\mu_o c_t r_e^2}{k}=\frac{750(4000)^2}{6.6667\times10^7}=180$$

Table 8
Reservoir and fluid properties of the synthetic case

Property	Value	Property	Value
t_p (hour)	300	B_0 (RB/STB)	1
h (ft)	100	c_t (psi^{-1})	3×10^{-6}
r_w (ft)	0.3	μ_0 (cp)	1
r_e (ft)	4000	ϕ	0.1
q_0 (STB/bbl)	500		

For this case, the average reservoir pressure is 4776.4489 psia and its actual value from simulation (test design) is 4776.99 psia. Using a and b, the parameters kh and V_p can be calculated as shown in Table 9. The comparison between the real and calculated values is also given in Table 10. Figure 3 graphically shows the accuracy of the LSM for a lot of data. Figure 4 shows the results of the actual values for this synthetic case. Because this example is a synthetic case, it is not required to use the trial and error method. The computation time for our method (LSM) is 0.011242 seconds.

Table 9
The calculated parameters

Parameter	Value
a (psia/hour)	-0.0075
b (psia)	32.4328
kh (md-ft)	1828.3961
V_p (bbl)	4.2791×10^9

Table 10
Comparison of parameters using our calculated method (LSM) and its actual value

	kh	V_p
Actual	2000	5.0265×10^9
Calculated	1828.3961	4.2791×10^9
Percentage error	-8.5802	-14.8692

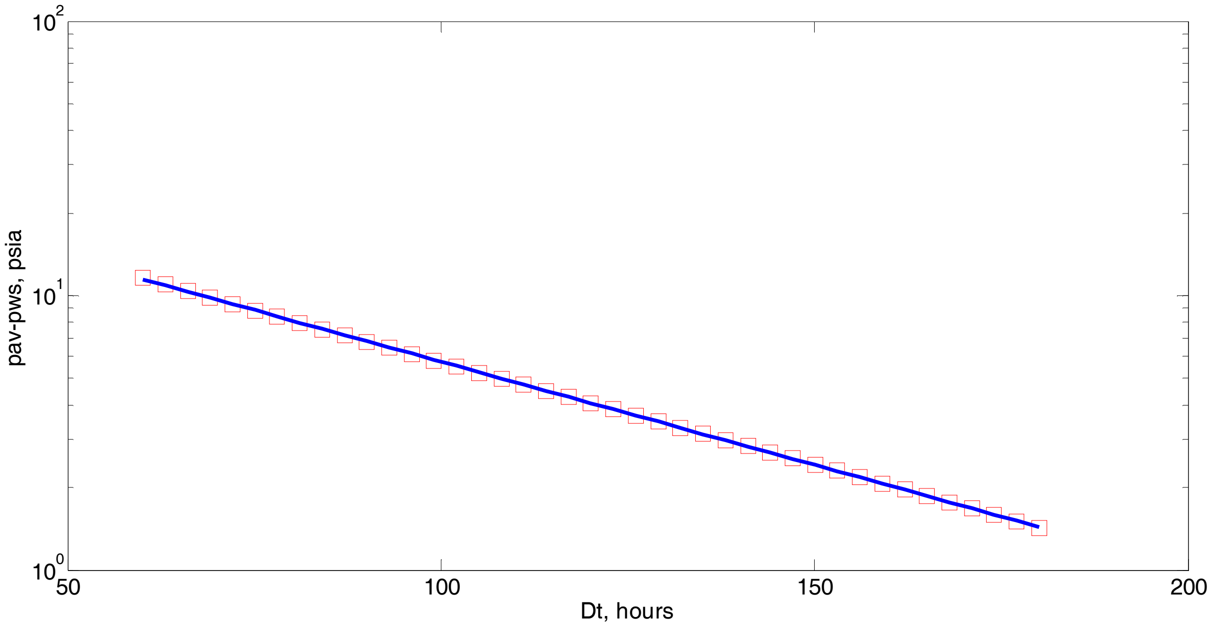

Figure 3
Data point and straight line obtained by least squares method for the synthetic case on semi-log scale

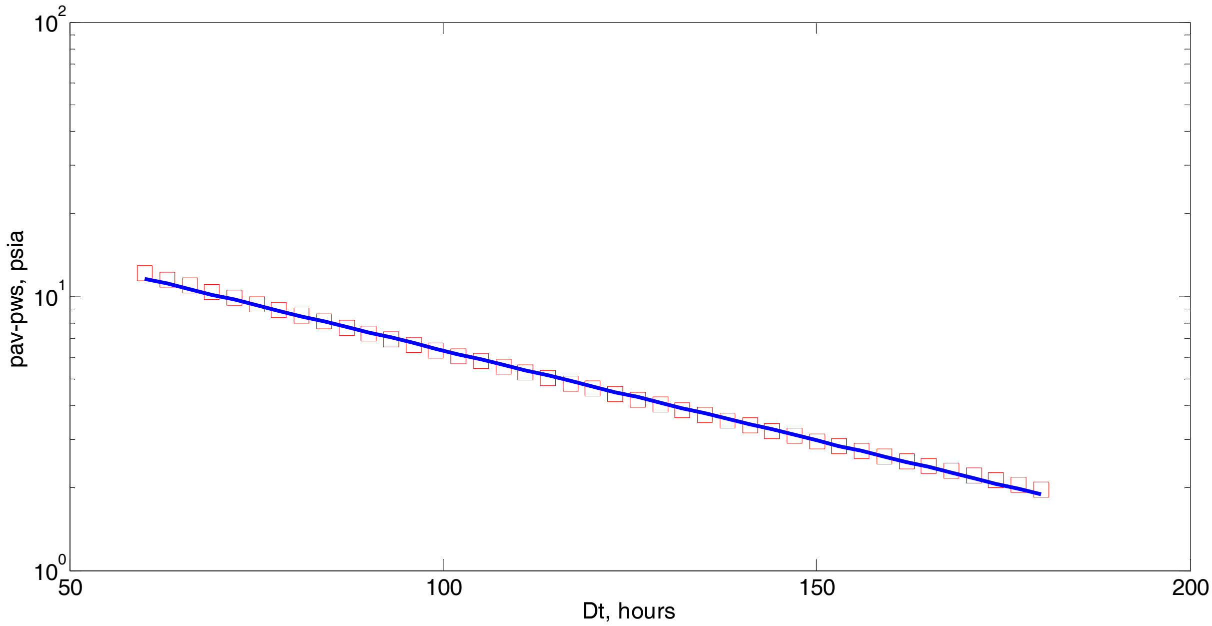

Figure 4
Data point and straight line of the actual values for the synthetic case on semi-log scale

6. Conclusions

This paper presented a new method based on the well-known LSM technique which reformulates and reinterprets the previous methods as a separable least squares problem. The LSM can be used in any other well testing methods rather than the mentioned method (modified Muskat method) in this paper. This method has some advantages over the other methods. As one of its main features, it can be noted that the method improves and increases the accuracy of calculations. In addition, the presented technique reduces the time of calculation and the amount of memory needed to compute and interprete data. Moreover, the numerical results suggest that the LSM should be capable of producing smooth interpretable estimates of reservoir parameters from data with an error smaller than the previous methods.

Nomenclature

a	: Slope of the straight line (psig/hour)
a_r	: Coefficients of Chebyshev functions
B_o	: Formation volume factor (res vol/surface vol)
b	: Intercept of the straight line (psig)
b_r	: Coefficients of Chebyshev functions
c_t	: Total isothermal compressibility factor (psi^{-1})
E	: Error function
h	: Net formation thickness (ft)
k	: Reservoir rock permeability (md)
N	: Number of given data
p_i	: Initial reservoir pressure (psi)
p_{ws}	: Shut-in BHP (psi)
$\overline{p}$	: Volumetric average or static drainage-area pressure (psi)
p^*	: MTR pressure trend extrapolated to infinite shut-in time (psi)

q_o	: Flow rate (stb/day)
r_e	: External drainage radius (ft)
r_w	: Wellbore radius (ft)
t_p	: Production time (hours)
Δt	: Time elapsed since shut-in (hours)
V_p	: Pore volume (drainage volume) (bbl)
μ_0	: Oil viscosity (cp)
φ	: Porosity
ψ	: Chebyshev functions

References

Ahmed, T., and Meehan, D. N., Chapter 1- Well Testing Analysis, Advanced Reservoir Management and Engineering (2nd Edition), p. 1-226, 2012.

Al-Attar, H. H., and Abdul-Majeed, G. H., A Simple Approach for Estimating Average Reservoir Pressure in Depletion-type Reservoirs, PETSOC Conference Paper, 1988.

Bjorck, A., Numerical Methods for Least Squares Problems, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, 1996.

Boyd, J. P., and Xu, F., Divergence (Runge Phenomenon) for Least-squares Polynomial Approximation on an Equispaced Grid and Mock-Chebyshev Subset Interpolation, Applied Mathematics and Computation, Vol. 210, No. 1, p. 158-168, 2009.

Chaudhry, A. U., Pressure Drawdown Testing Techniques for Oil Wells, Oil Well Testing Handbook, p. 107-152, 2004.

Crump, J. G., and Hite, R. H., A New Method for Estimating Average Reservoir Pressure, The Muskat Plot Revisited, SPE Journal Paper, 2008.

Earlougher, R. C. Jr., Advances in Well Test Analysis, Monograph Series, SPE, Dallas, 1977.

Kuchuk, F. J., Onur, M., and Hollaender, F., Deconvolution, Developments in Petroleum Science, Vol. 57, No. 4, p. 115-195, 2010.

Larson, V. C., Understanding the Muskat Method of Analyzing Pressure Buildup Curves, Journal of Canadian and Petroleum Technology, Vol. 2, p. 136-141, 1963.

Lee, W. J., Well Testing, Textbook Series, SPE, Dallas, 1982.

Li, B., and Jia, Z., Some Results on Condition Numbers of the Scaled Total Least Squares Problem, Linear Algebra and Its Applications, Vol. 435, No. 3, p. 674-686, 2011.

Markovsky, I., and Huffel, S. V., High-performance Numerical Algorithms and Software for Structured Total Least Squares, Journal of Computational and Applied Mathematics, Vol. 180, No. 2, p. 311-331, 2005.

Matthews, C. S., Russell, D. G., Pressure Buildup and Flow Tests in Wells, Monograph Series, SPE, Dallas, 1967.

Pollard, P., Evaluation of Acid Treatment from Pressure Build-up Analysis, AIME, Vol. 216, p. 38-43, 1959.

Van Everdingen, A. F. and Hurst, W., The Application of the Laplace Transformation to Flow Problems in Reservoir, AIME, Vol. 1, No. 12, p. 305-324, 1949.

Verga, F., Viberti, D., and Borello, E. S., A New Insight for Reliable Interpretation and Design of Injection Tests, Journal of Petroleum Science and Engineering, Vol. 78, No. 1, p. 166-177, 2011.