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Investigating the Effect of Heterogeneity on Buckley-Leverett Flow Model

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Abstract

The performance of water flooding can be investigated by using either detail numerical modeling or simulation, or simply through the analytical Buckley-Leverett (BL) model. The Buckley-Leverett analytical technique can be applied to one-dimensional homogeneous systems. In this paper, the impact of heterogeneity on water flooding performance and fractional flow curve is investigated. First, a base one-dimensional numerical model is considered and then the numerical model is made and validated through comparison with the Buckley-Leverett fractional flow equation. Then, the model is extended to two dimensions and heterogeneity is incorporated in the modeling by using six different heterogeneous models. In particular, distributions for permeability values are considered. Fractional flow curves and water flooding performances are investigated for each individual model. A modification in the Buckley-Leverett fractional flow equation is discussed in order to consider the heterogeneity effects.

Keywords: Water Flooding, Buckley-Leverett, Fractional Flow, Saturation Distribution, Numerical Modeling

1. Introduction

Heterogeneity is one of the most important factors that affect reservoir recovery performance. One of the most important problems in reservoir management and enhanced oil recovery (EOR) applications is to describe inter-well heterogeneity for optimum well spacing and process design (Chopra et.al., 1990). Incorporating a suitable level of heterogeneity into reservoir simulations is necessary for accurate prediction of production rates and final recoveries. The spatial correlation of petrophysical properties, particularly permeability extrema, exerts a profound influence on flow underlying reservoir displacement and depletion processes. Common modeling techniques are founded on Gaussian assumptions for statistical distributions. Such Gaussian-based approaches can inadequately model the permeability extrema that can dominate reservoir performance. Since the microstructures of porous media are usually disordered and extremely complicated, this makes it very difficult to analytically find the permeability of the media, especially for unsaturated (or multiphase) porous media (Gaynor et.al., 2000).

Conventionally, the permeabilities of porous media were found by experiments (Levec et.al., 1986). Furthermore, much effort was also devoted to numerical simulations of permeabilities of porous media. Simacek and Advani (1996) performed numerical solutions by reducing a two-dimensional problem to a one-dimensional equation. Adler and Thovert (1998) applied a fourth-order finite

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difference scheme to calculating the permeabilities of real Fontainebleau sandstone. Ngo and Tamma (2001) employed the finite-element method to calculate the permeability of the porous fiber mat by assuming the Stokes flow in the inter-tow region and the Brinkman's flow inside the tow region. Compared with single-phase (or saturated) flow in porous media, the multiphase (or unsaturated) immiscible flows in porous media are not well understood. The multiphase immiscible flows in porous media are very important in practical applications such as the petroleum industry, chemical engineering, and soil engineering. The Lattice Boltzmann Method (LBM) (Benzi et al., 1992; Sahimi and Mukhopadhyay, 1996; Martys and Chen, 1996; Chen and Doolen, 1998), based on the Navier-Stokes equation coupled with Darcy's law, has been extensively used to simulate multiphase flows through porous media in order to understand the fundamental physics associated with enhanced oil recovery, including relative permeabilities. The LBM is particularly useful for complex geometrical boundary conditions and varying physical parameters.

A permeable media is heterogeneous if there is spatial variation in its properties. Reservoir heterogeneity is an important parameter in reservoir performance prediction, and describing the heterogeneity is an important step in reservoir identification. Spivak (1974) and Giordano et al. (1985) included random heterogeneity generated by using a normally distributed permeability field without spatial correlation. Moissis et al. (1989), Araktingi et al. (1988), and Waggoner et al. (1992) used geostatistical techniques to generate permeability fields based on the coefficient of permeability variation, or the Dykstra-Parson coefficient, and autocorrelation. Sorbie et al. (1994) extended Waggoner et al.'s work (1992) to define flow regimes and their significance for small autocorrelation.

There are two ways for indicating main fluid flow equations: 1- pressure method, 2- fractional flow method. The pressure method is widely used in hydrology. In this method, the main equations are written in terms of each of the two phase pressure that is obtained by directly substituting of Darcy equation in material balance equation of the phases. The fractional flow method is more applicable in petroleum engineering. This method has two equations, namely pressure equation and saturation equation. The pressure equation is an elliptical equation that is solved for overall flow and pressures. The saturation equation is of advection-diffusion form with hyperbolic properties which describe progressive front velocity. In the absence of capillary and gravity forces, the saturation equation could be solved analytically by the Buckley-Leverett equation.

The Buckley-Leverett model is applicable to homogeneous reservoir analysis. Higher reservoir heterogeneity results in higher fractional flow of water. A method to include the heterogeneity effects in the Buckley-Leverett fractional flow equation was discussed against various heterogeneous reservoir models through a defined parameter H. The two-dimensional numerical solution using the ECLIPSE software is verified by comparing with the analytical solution of the two-dimensional Buckley-Leverett model. An averaged saturation value at each point was considered in the two-dimensional reservoir model in order to reduce it to a one dimensional model and to apply the simple BL analytical model. Based on this primary and validated model, various heterogeneous models are investigated assuming that the average permeability of these models is equal to the permeability of the homogeneous model, and then the effect of each model on the water flooding performance is investigated. Two types of heterogeneous models are considered: 1) three models of two-part reservoirs, and 2) two models of reservoirs with permeability distribution. All of the models are investigated and the results are shown and discussed.

2. Assumptions

The following assumptions are made:

- The system is linear, horizontal, and of constant thickness;

- The flow is isotherm and incompressible and obeys Darcy's law;
- The displacement is piston like (the mobility ratio is less than 1);
- Gravity and capillary forces are negligible;
- The relative permeability obeys the Brooks-Corey equation of the following expressions:

$$S_{wn} = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}} \quad (1)$$

$$k_{rw} = S_{wn}^m \quad (2)$$

$$k_{ro} = (1 - S_{wn})^n \quad (3)$$

Table 1
Parameters of Brooks-Corey relative permeability correlation.

m	***n***	S_{wi}	S_{or}	k_{rw}	k_{ro}
4	4	0.1	0	1	1

- The reservoir rock is neutral (or mix) wet.
- Rock compressibility is neglected; therefore, rock expansion drive mechanism does not exist. Reservoir oil and injection water properties at 1500 psi are shown in the following table.

Table 2
Fluids properties of the model.

Phase	Viscosity (cP)	Density (lb/ft^3)	Formation Volume Factor (rbbl/stb)
Oil	3.1527	49	1.01
Water	1	63	1

3. Numerical model description

3.1. One-dimensional model

The one-dimensional model is constructed in Cartesian coordinates. The number of grids in the model is 40×1×1 in X, Y, and Z direction. The producer is located at the last block and is controlled by BHP of 1500 psi. The injector is located at the first block.

3.1.1. Numerical solution

The simulator (ECLIPSE 2009.1) does not directly compute fractional flows. However, by the use of the outlet flow of each block in X direction, fractional flow could be computed through the following equation:

$$f_{w,n} = \frac{q_w}{q_w + q_o} = \frac{bflowi}{bflowi + bflooi} \quad (4)$$

where, *bflooi* is the oil outlet flow rate of each block in X direction and *bflowi* is the water outlet flow rate of each block.

Figure 1 shows that the numerical one-dimensional model is acceptable. Front location (x) at a specified time is calculated through the following equation:

$$x = \left.\frac{df_w}{dS_w}\right|_{S_w} \times (5.615 \times q \times t / A \times \varphi) \qquad \text{(field units)} \tag{5}$$

By fitting a function to f_w curve, the derivative $\left.\frac{df_w}{dS_w}\right|_{S_w}$ can be calculated. Dimensionless distance is defined as the ratio of the *x* divided by total length of the reservoir.

3.1.2. Validation of the numerical homogeneous one-dimensional model

In order to validate the numerical model, the numerical solution should be compared with the analytical solution which uses the BL fractional flow equation.

a. Analytical solution

The fractional flow equation, once neglecting the gravity and capillary forces, is as follows (Dake, 1978):

$$f_{wbl} = \frac{1}{1 + \frac{1}{M}}, \qquad M = \frac{\lambda_w}{\lambda_o} = \frac{k_{rw}/\mu_w}{k_{ro}/\mu_o} \tag{6}$$

where, *M* is the mobility ratio. Substituting K_{rw} and K_{ro} by equations 2 and 3 gives:

$$f_{wbl} = \frac{1}{1 + 0.315177 \times (\frac{(1 - S_{wn})^4}{S_{wn}^4})} \tag{7}$$

Comparison of the numerical and analytical solutions at a point before the water breakthrough, for example at 0.29 PV_{inj}, is shown in Figure 1. It is noticeable that the partial difference between analytical and numerical solutions of f_w occurs because the water saturation is suddenly reduced in the displacement front.

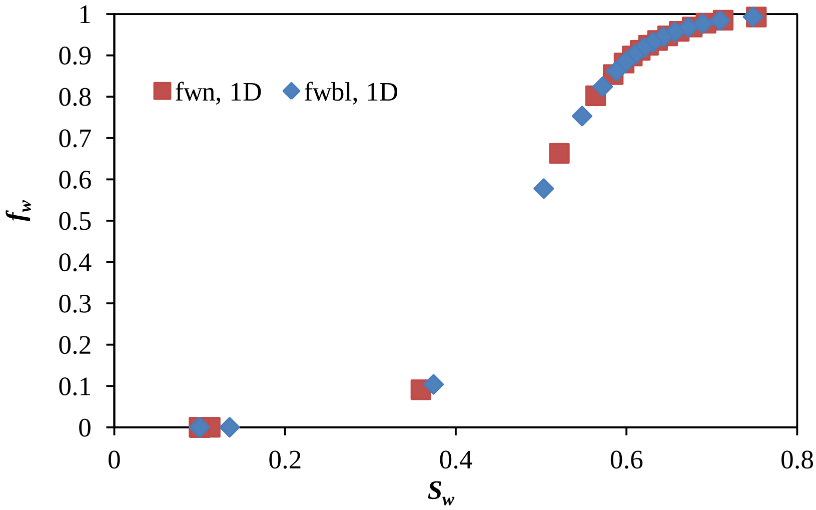

Figure 1
Water fractional flow of the one-dimensional homogeneous model (comparison of numerical and analytical solutions).

Figure 2 shows the validity of the numerical one-dimensional model. Comparison of the saturation distribution versus the dimensionless distance at point 0.29 PV_{inj} is shown in Figure 2. As shown in Figure 2, there is a difference in the saturation distribution of these two solutions in the front of the displacement; the area between numerical and analytical solutions is 0.06. For the analytical solution result, it is assumed that water saturation is suddenly reduced from front water saturation to irreducible water saturation in the front of the displacement, while in the numerical solution, the saturation profile is determined through saturation equations solution for each time step and the displacement front is not a line but a profile of variable water saturation as shown in Figure 3.

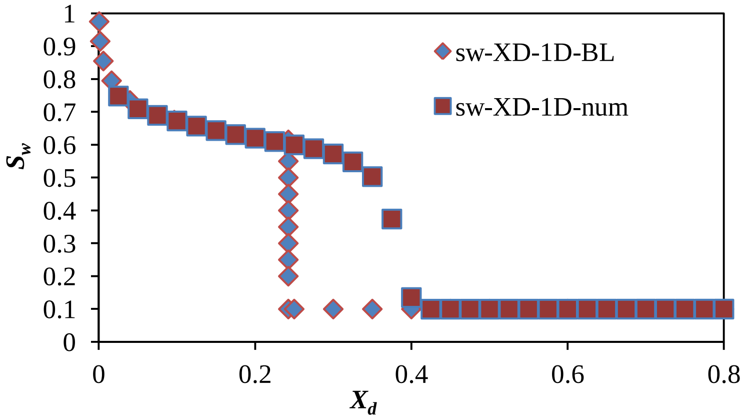

Figure 2
Saturation distribution of the one-dimensional homogeneous model at 0.29 PV_{inj} of water.

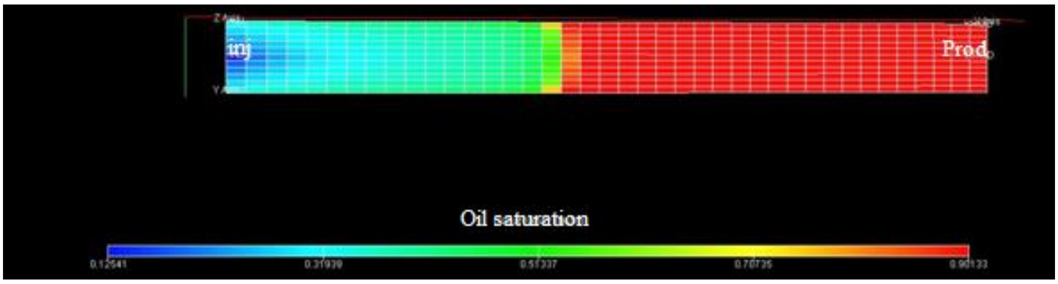

Figure 3
An example of the displacement front profile of the numerical model.

3.2. Two-dimensional model

Three numerical models of different number of grids are considered. As it is shown in Figure 4, the model and the numerical solution are not sensitive to the grid number. Therefore, we consider a homogeneous two-dimensional numerical model of 40×10×1 grids for convenience.

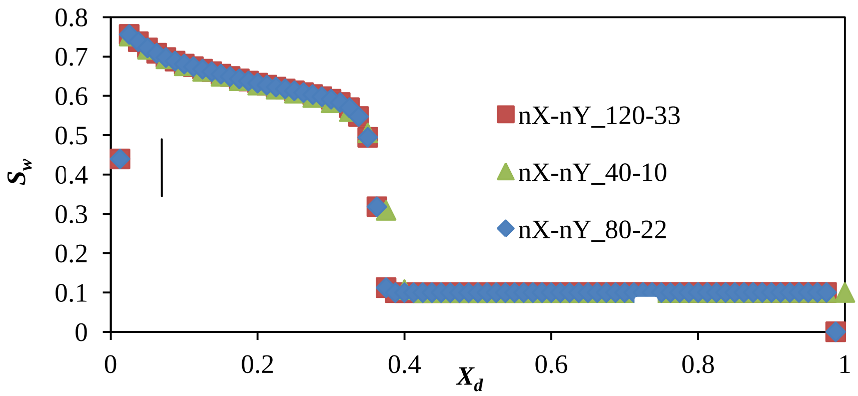

Figure 4
Sensitivity of grid number of the model.

3.2.1. Numerical solution

For the numerical solution, 40 regions are defined that consist of blocks of the same X value and 1 to 10 Y values. Then, the water saturation region is asked by the simulator for reporting purposes. This is repeated for the block outlet flow rate in the X direction. Then, the fractional flow of each region is calculated by using equation 4.

3.2.2. Validation of two-dimensional numerical model

The same approach as discussed in the one-dimensional model is used for the two-dimensional model validation.

a. Analytical solution

Since the permeability of the reservoir is constant, the two-dimensional model was reduced to a one-dimensional model. In the analytical solution, the average block saturation of the same X value and 1 to 10 Y values for all of the X values was determined. The analytical solution of this two-dimensional model was the same as that of the one-dimensional model. The results are shown in Figures 5 and 6.

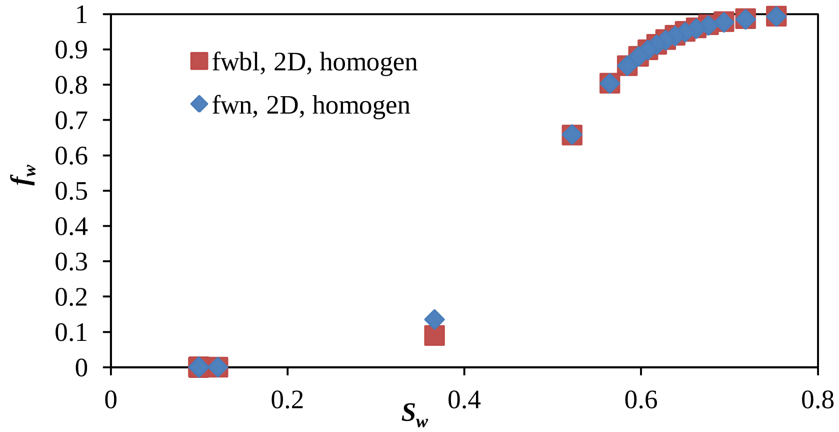

Figure 5

Fractional flow of two-dimensional homogeneous model (comparison of numerical and analytical solution).

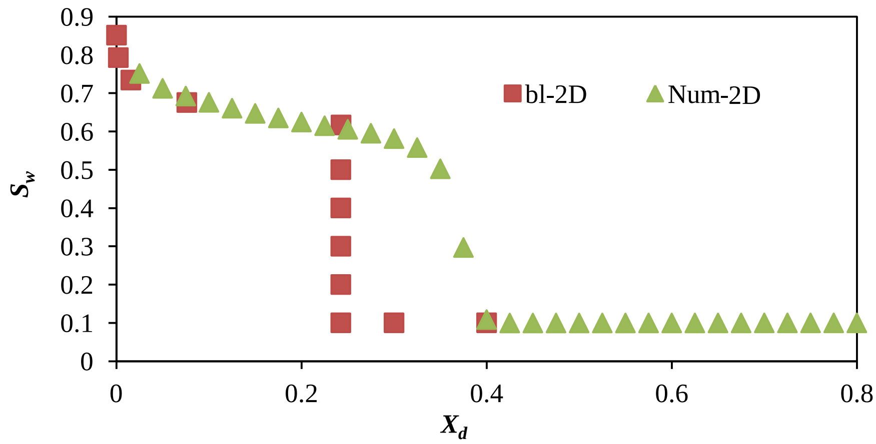

Figure 6

Saturation distribution of two-dimensional homogeneous model (comparison of numerical and analytical solution).

The area between numerical and analytical solution is 0.06, the same as that of the one-dimensional model. Figure 5 indicates that the numerical solution for the two-dimensional homogeneous model is very close to the analytical solution by considering the average saturation, confirming the validation of the results. Now, one can rely on the numerical two-dimensional results and can investigate the effects of heterogeneity.

3.3. Two-dimensional heterogeneous models

In this section, heterogeneity is incorporated into the model. If the absolute permeability of the medium depends on the position x, the hyperbolic Buckley-Leverett equation cannot be a satisfactory model. In contrast to the homogeneous case, a diffusion term has to be included in order to obtain a well-behaved model i.e. to overcome the instability of the Buckley-Leverett equation in a heterogeneous medium. In the considered model, the pressure gradient and viscosity are assumed constant. Then, for including a diffusion term, a coefficient should be adjusted for the relative permeability in the BL fractional flow equation. This instability in the BL solution is due to permeability heterogeneity and it seems that the fractional flow of water is increased by increasing the heterogeneity. Then, the parameter that shows the heterogeneity of the model is defined as H. As a result, the coefficient of relative permeability should be expressed as the inverse of heterogeneity parameter, i.e. $1/H$. The modified fractional flow equation for heterogeneous media would be of the following form, based on Ehsan Kamari et al. (2012):

$$f_w = \frac{1}{1 + (\frac{1}{H} \times \frac{1}{M})} \quad (8)$$

3.3.1. Two-part model

In this model, the reservoir is divided into two parts in Y direction. Three versions of this type are considered:

a. Layers with the same width

The two layers have different permeability distributions by an average of 500 md. Other rock and fluid properties are the same as that of the previous homogeneous two-dimensional model. The comparison of the fractional flow curves of these three considered cases is shown in Figure 7. The modified fractional flow equation and the relative permeability coefficient for each case are provided in Table 3.

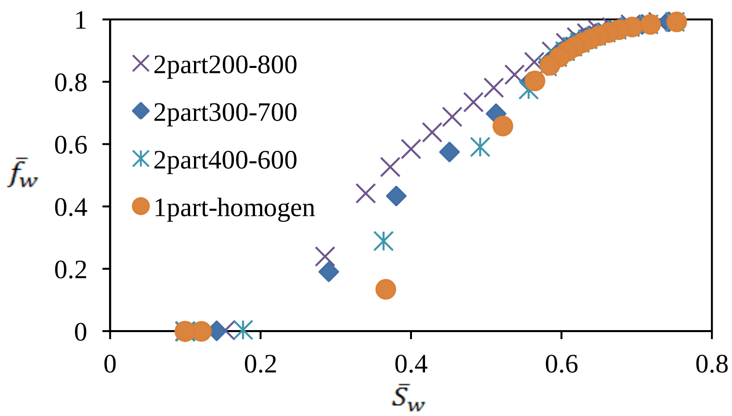

Figure 7
Fractional flow of parts of equal cases of the two-part model at 0.29 PV_{inj} of water.

Table 3
H value of parts of equal cases of the two-part model.

Cases	200,800	300,700	400,600
***H* value (equation 8)**	6.925	2.605	1.313

The error of saturation averaging in the solution process for one of the cases in this model is shown in Figure 8. It shows that the average error has the higher value in the front of the displacement.

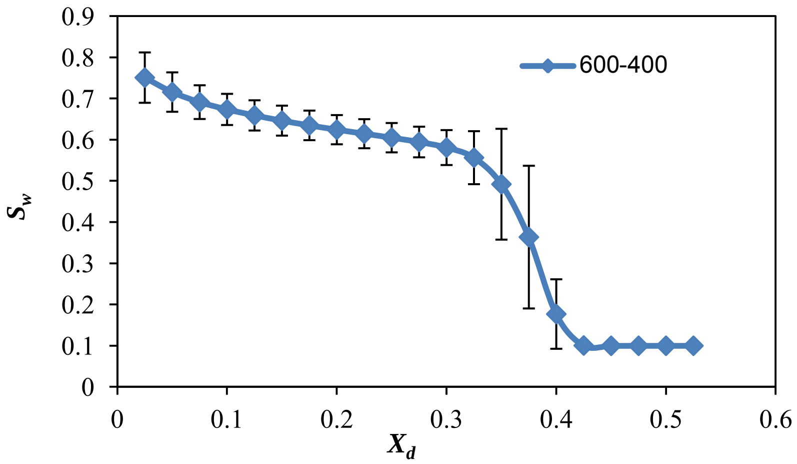

Figure 8
Saturation averaging error (saturation versus dimensionless distance).

b. Layers with different widths

In this model, a two-part reservoir of unequal widths of $b/3$ and $2b/3$ is considered, where b is the total width of the reservoir. The permeability of each part is chosen in such a way that the overall average of the permeability distribution becomes equal to 500 md. The number of grids is 40×12×1. Three cases are provided in this type. Figure 9 shows the comparison of the fractional flow curve of these cases and the homogeneous model. The modified fractional flow equation and the relative permeability coefficient for each case are shown in Table 4.

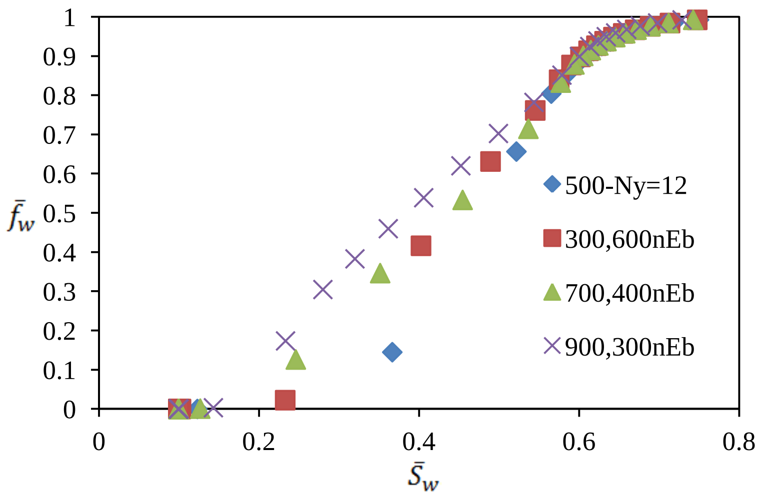

Figure 9
Fractional flow of unequal width cases of the two-part model at 0.29 PV_{inj} of water.

Table 4

H value of unequal cases of the two-part model.

Cases (*b*/3, 2*b*/3)	300,600	700,400	900,300
***H* value (Equation 8)**	1.733	1.793	6.269

c. Parts of equal value of permeability multiplied by width

Four cases are considered in this type of reservoir model, where the values of permeability multiplied by width id are equal for the two parts because the average permeability is 500 md. Figure 10 shows the comparison of the fractional flow curves of these cases and the homogeneous model. The relative permeability coefficient for each case is provided in Table 5.

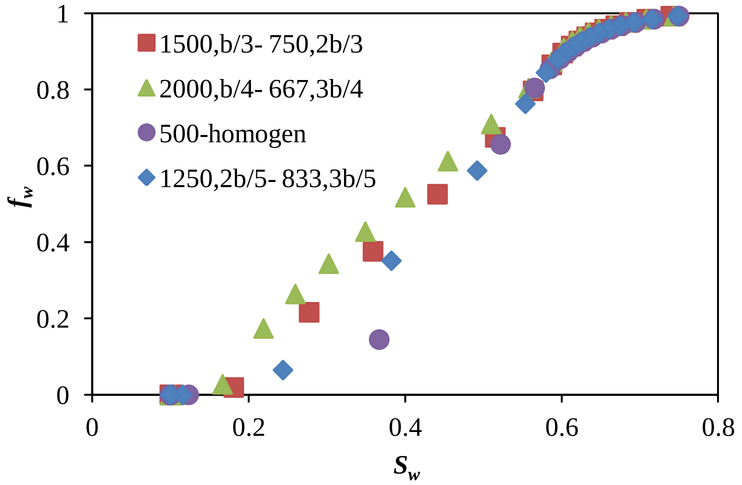

Figure 10

Fractional flow of cases of equal value of permeability multiplied by width of the two-part model at 0.29 PV_{inj} of water.

Table 5

H value of cases of equal value of permeability multiplied by width in the two-part model.

Cases (k;w)	1500; *b*/3,750; 2*b*/3	2000; *b*/4,667; 3*b*/4	1250; 2*b*/5,833, 3*b*/5
H value (Equation 8)	2.25	5.87	1.39

3.3.2. Reservoir models with permeability distribution

In these models, the permeability heterogeneity is provided by considering specific distributions for the permeability of the models. Two types of distributions for permeability values are considered.

a. Normal distribution

The number of the grids in this model is 40×11×1. The permeability of the model follows a normal distribution. Five cases of constant average of 500 md and various standard deviations are considered. As shown in Figure 11 there is no significant difference between the results of the homogeneous model and these examined models.

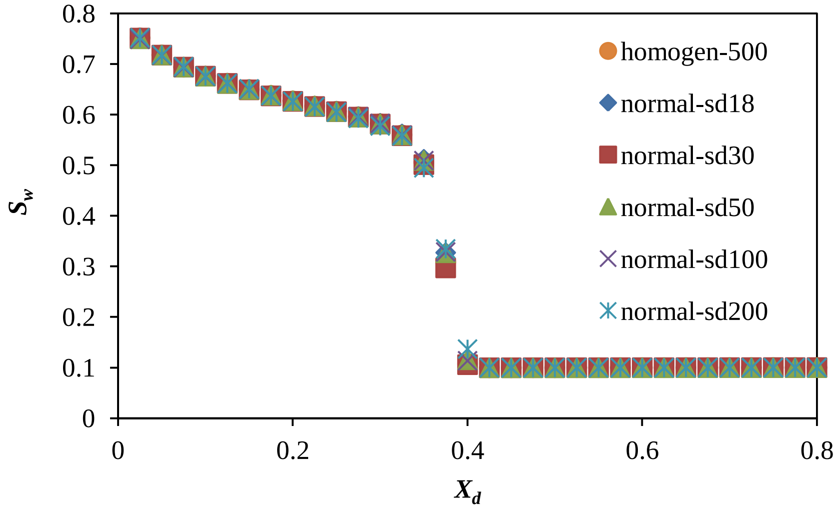

Figure 11
Saturation distribution of reservoir models with a normal permeability distribution at 0.29 PV_{inj} of water.

The cumulative production of the different reservoir models in these cases is shown in Figure 12.

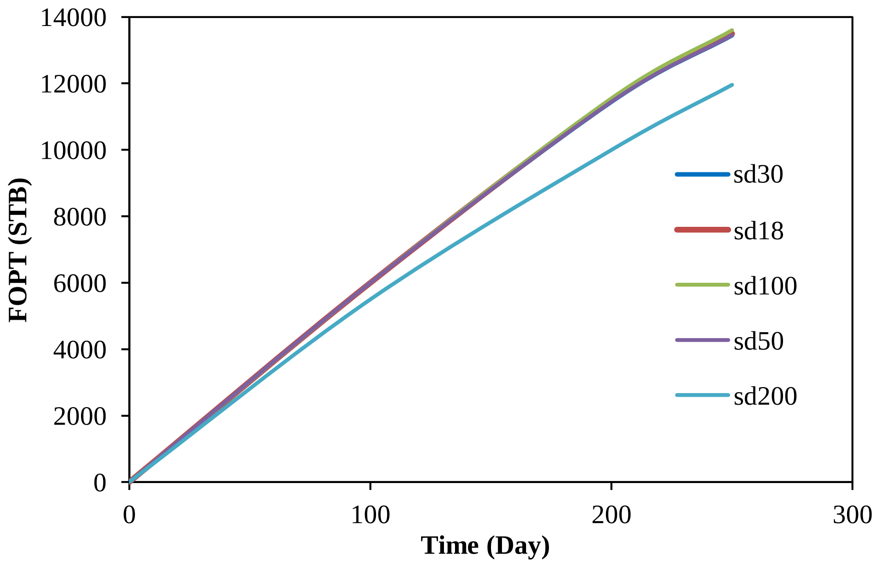

Figure 12
Cumulative oil production of normal permeability distribution cases.

b. Lognormal distribution

The number of the grids is the same as the previous model. Permeability has a lognormal distribution of an average value of 500 md but at various standard deviations. Figure 13 shows the comparison of the fractional flow curve of these cases and the homogeneous model. The modified fractional flow equation and the relative permeability coefficient for each case are shown in Figure 13. The *H* values are provided in Table 6, and the cumulative oil production versus time for different cases of heterogeneous reservoir model with lognormal permeability distribution is shown in Figure 14.

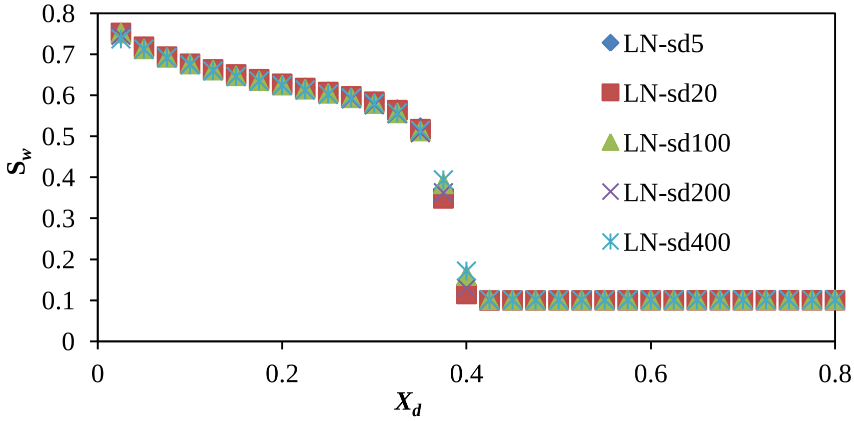

Figure 13
Saturation distribution of lognormal permeability distributions cases at 0.29 PV_{inj} of water.

Table 6
H value of cases of lognormal permeability distribution.

Cases (standard deviation)	100	200	300
***H* value (Equation 8)**	1.033	1.087	1.156

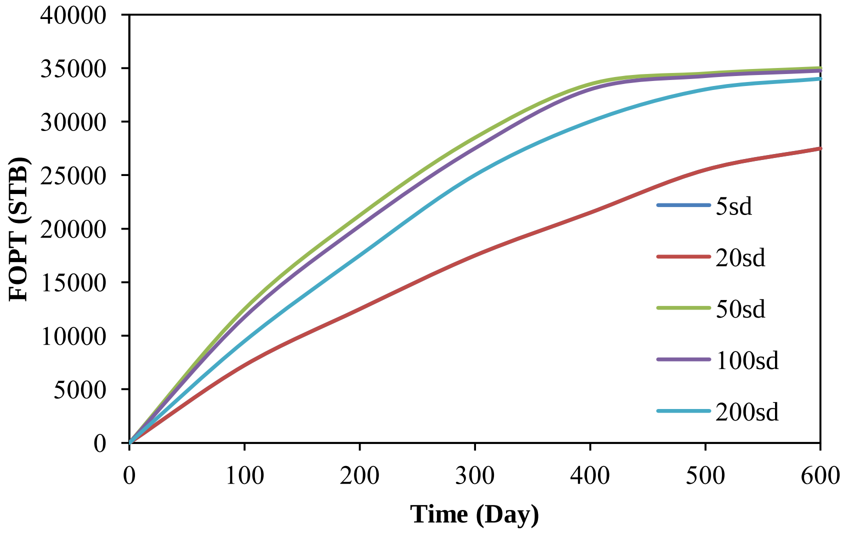

Figure 14
Cumulative oil production of lognormal permeability distributions cases.

The difference between various cases is justified by their histogram, which is shown in Figure 15 for cases with a lower standard deviation (e.g. 5 and 20 md); permeability is less scattered around 500 md. Its result is therefore close to that of the homogeneous model. In cases with a greater standard deviation, data are more scattered and the cumulative production has higher values since the lognormal distribution includes very low and very high permeability values. As shown in Figure 15, the case of 400 md standard deviation contains very low permeability values and thus its cumulative production is lower than the others.

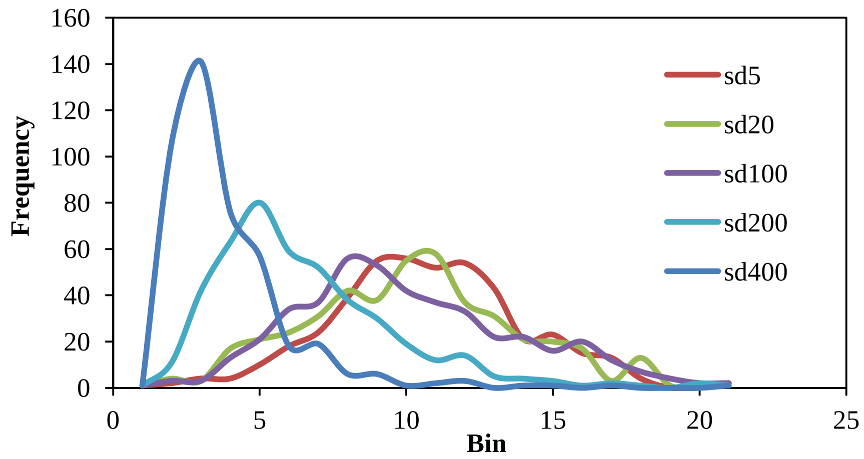

Figure 15
Histograms of lognormal permeability distribution cases.

4. Conclusions

The Buckley-Leverett model is applicable to homogeneous reservoirs. Higher reservoir heterogeneity results in a higher fractional flow of water. A method to include the heterogeneity effects in the Buckley-Leverett fractional flow equation was discussed against various heterogeneous reservoir models through a defined parameter *H*. A two-dimensional numerical solution using the ECLIPSE software was verified by comparing with an analytical solution of the two-dimensional Buckley-Leverett model. An averaged saturation value at each point was considered in the two-dimensional reservoir model in order to reduce it to a one-dimensional model, and to use a simple BL analytical model. Based on this primary and validated model, various heterogeneous models were investigated assuming that the average permeability of these models was equal to the permeability of the homogeneous model. The following specific results were found:

1. In the investigated two part models, heterogeneity is related to the width of each part and its permeability. It seems that by increasing *H* (i.e. heterogeneity degree), the fractional flow of water increased;
2. Cumulative oil production is reduced by increasing standard deviation for cases of normal distribution;
3. In the case of low standard deviation lognormal permeability distribution, the cumulative oil production had a low deviation compared to the homogeneous model results, provided that the standard deviation is small; however, for the cases of higher standard deviation values, changes in cumulative oil production trend are justified by the histogram of distributions.

Nomenclature

A	: Area [ft^2]
bflowi	: Water flow rate of block *i* [rbbl]
bflooi	: Oil flow rate of block *i* [rbbl]
FOPT	: Total field oil production [stb]
$f_{w,\mathrm{n}}$	: Water fractional flow
H	: Heterogeneity parameter

K	: Permeability [md]
k_{ro}	: Oil relative permeability
k_{rw}	: Water relative permeability
M	: Mobility ratio
m,n	: Brooks-corey relative permeability parameters
PV_{inj}	: Injected pore volume
q_o	: Oil flow rate[stb]
q_w	: Water flow rate [stb]
rbbl	: Reservoir barrel
S_{or}	: Residual oil saturation
S_w	: Water saturation
S_{wi}	: Irreducible water saturation
S_{wn}	: Normalized water saturation
t	: Time [day]
W	: Width [ft]
x	: Front location [ft]
X_d	: Dimensionless front location
λ_w	: Water mobility [md/cp]
λ_o	: Oil mobility [md/cp]
μ_o	: Oil viscosity [cp]
μ_w	: Water viscosity [cp]

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