Comparing Geostatistical Seismic Inversion Based on Spectral Simulation with Deterministic Inversion: A Case Study

Hamid Reza Ansari\textsuperscript{1}, Reza Motafakkerfard\textsuperscript{1,*}, and Mohammad Ali Riahi\textsuperscript{2}

\textsuperscript{1} Department of Petroleum Exploration, Petroleum University of Technology, Abadan, Iran
\textsuperscript{2} Institute of Geophysics, University of Tehran, Tehran, Iran

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Abstract
Seismic inversion is a method that extracts acoustic impedance data from the seismic traces. Source wavelets are band-limited, and thus seismic traces do not contain low and high frequency information. Therefore, there is a serious problem when the deterministic seismic inversion is applied to real data and the result of deterministic inversion is smooth. Low frequency component is obtained from well log data; however, but when well log and seismic data are used together, it faces a problem which is a function of the support of scale of measurements. Well log data have a high vertical resolution while seismic data represent low details in vertical direction.

Geostatistical seismic inversion (GSI) is a method to overcome the aforementioned limitations. GSI uses well log and seismic data together in the geostatistical frameworks. In this study, a new approach of geostatistical inversion based on spectral geostatistical simulation is used. This approach is performed in frequency domain and stochastic framework. Distinct from sequential simulation, spectral simulation method is a direct method, which does not require an acceptance/rejection step. Hence, GSI algorithm based on spectral simulation is fast. This approach is performed in a case study of an Iranian gas field in the Persian Gulf basin. The upper-Dalan and Kangan are two main formations of this field. The results of GSI are compared with deterministic inversion and it is concluded that, as opposed to deterministic inversion, GSI can recover low frequency components.

Keywords: Geostatistical Seismic Inversion, Deterministic Inversion, Spectral Simulation, Geostatistics

1. Introduction
Acoustic impedance (AI) is an important rock property that can be obtained from seismic data during seismic impedance inversion. Most of seismic inversion methods are based on minimizing differences between synthetic seismic and real seismic responses. Synthetic seismic responses are the result of convolution of wavelet and earth reflectivity. Earth reflectivity is a rock property, which is a function of acoustic impedance. The inversion methods which operate in minimizing error are known as “deterministic inversion.”

Deterministic seismic inversion such as sparse spikes or model-based inversion is smooth in results, which is due to its limitations. Francis discussed some of these limitations (Francis, 2006). The significant limitation is missing low frequency information due to the band-limitation of real seismic data. Since the source wavelet is band-limited and does not cover all frequencies, low and high

* Corresponding Author:
Email: r_motafakker@put.ac.ir
frequency components are hidden in the seismic responses. Missing the low frequency is important due to the fact that low frequency components contain critical information about the absolute impedances values (Francis, 2010). Low frequency information can be obtained from well log data. For adding log data to seismic data, one may encounter a serious problem, which is known as the support of scale measurement of data. Well data have a high vertical resolution versus the seismic data. In deterministic methods, the scale-up of well data to larger support measurements is used. Scale-up is an averaging method, which reduces variability of measurements, when they are scaled up to larger supports. To overcome these problems, seismic data in a geostatistical framework are inverted. Geostatistical seismic inversion (GSI) is a method introduced to improve deterministic inversion results.

GSI was introduced and tested by Haas et al. (Haas et al., 1994) for the first time in 1994. They employed a sequential Gaussian simulation (SGS) to produce impedance realizations in inversion process. In each grid node, a random large trace realization from seismic and log data is generated and then the best trace, which becomes data conditioning, is selected. Grijalba-Cuenca et al. offered another GSI algorithm, which worked grid by grid cell, instead of trace by trace (Grijalba-Cuenca et al., 2000). This algorithm estimated a local probability density function (PDF) from the PDF of the available control points by a kriging technique. The stratigraphic and structural information is incorporated in this method. This information is available in the form of time horizons. The final result is selected in simulated annealing method. Simulated annealing and SGS perform in an accepting or rejecting stage. Francis offered a new method of geostatistical inversion based on spectral simulation (Francis, 2005). Spectral simulation is performed in frequency domain and its main advantage is its fast run-time, due to the fact that a global density spectrum is calculated once and the inverse Fourier transform is performed only once to generate a realization (Yao et al., 2004).

Recovering absolute impedance values is important when we should detect the thin bed (Zhang et al., 2012 and Merletti et al., 2003) or obtain other reservoir properties by high accuracy. In this study, at first, the deterministic inversion method is performed on a carbonate field from Iran and then a geostatistical seismic inversion based on spectral simulation (Francis, 2010 and Francis, 2005) is accomplished in this field.

2. Geological setting

This paper is focused on a portion of an Iranian gas field in the Persian Gulf basin. The structure of this field is dome shaped, which has a gentle dip on the flanks. Kangan (Triassic) and upper Dalan (Permian) are two main formations in this field; each formation is divided into two different layers. From top to bottom, K1, K2, K3, and K4 are four reservoir layers in this field (Figure 1). K2 and K4 are two main gas reservoirs (Tavakoli et al., 2011). This field is a heterogeneous carbonate-evaporate reservoir in which dolomite, limestone, and anhydrite are the key lithology of the formations.

The available data sets of this study belong to four wells and 3D post-stack seismic data. Well logs data consist of sonic and density logs, which are used to construct acoustic impedance log (Figure 2). Moreover, three interpreted time horizon surfaces termed Dashtak-S7, K1, and K4, are available in these data sets.
3. Data preparation

The seismic data quality of this field is generally good over the entire time range. The sampling interval in the horizontal directions (inline and cross-line) is 1 ms and in the vertical direction is 4 ms. Bin sizes of 25 m/line (in the inline direction) and 6.25 m/line (in the cross-line direction) are used to acquire the seismic data.

Tying well log to seismic data is a primary procedure in all interpretation projects. The well data is converted from depth domain into time domain in this step. The depth to time model in the well location is provided by checkshot data. Then, the density and sonic logs will be combined into impedance and reflectivity log. The synthetic seismogram is generated by convolving the reflectivity and wavelet. A wavelet must be extracted from the seismic data. Statistical wavelet extraction is a common method, when a few pieces of well data are accessible (Edgar and Van der Baan, 2011). A wavelet is defined by amplitude and phase spectra. In statistical wavelet extraction, the phase spectrum is not computed and must be defined as a known parameter. The spectral time analysis of this seismic data evidences a zero phase spectrum. Thus, a zero phase wavelet is extracted from the seismic data alone in a statistical method. Then, the synthetic seismic traces are built in each well location. The cross correlation between synthetic and real seismic is averagely 0.74 (Figure 3). Figure 4 demonstrates the final wavelet extracted in the frequency domain. As it is shown, the wavelet is band-limited in frequency domain, and hence seismic data are filtered in a band-limited frequency bound; therefore, high and low frequencies are lost in seismic data.

In each geostatistical study, variogram (or other spatial relationship) analysis is performed in initial step. Stationarity is the main assumption to calculate variograms. Thereby, the seismic and well data must be trendless in variogram analysis. The two graphs in Figure 5 illustrate well data, in which one is the original graph, containing a simple linear trend, and the other is the de-trended data.
Figure 2
Well logs data for well B: DT (sonic), RHOB (density), and acoustic impedance versus time (milliseconds).

Figure 3
The synthetic seismogram and real seismic traces are shown at well B location; the cross-correlation coefficient is 0.747078.

Then, the well log data are used in vertical variogram analysis. Simple linear trend in log data can be removed by subtracting a least square fit straight line. In seismic data, the closet seismic attribute map
that does not contain trend is relative acoustic impedance. Seismic colored inversion (Lancaster et al., 2000) is a process that converts seismic data into relative acoustic impedance (Figure 6). Relative acoustic impedance is used in horizontal variogram analysis. If there is an anisotropic spatial correlation in variogram parameters in different directions, then the anisotropic variogram model must be used to interpolate the well data.

Figure 4
Average wavelet estimated is displayed in the frequency domain.

Figure 5
(a) Originally AI log data of four wells with trend and (b) de-trended AI log data in time domain.

4. Deterministic seismic inversion

Model-based inversion is currently the most popular method to integrate impedances by inverting seismic data. Generalized linear inversion (GLI) is a kind of model based on inversion that is used in this study as a deterministic inversion method. If it is assumed that $M$ is a vector of model parameter and $T$ is a data vector, then the relationship between them reads:

$$ T = F(M) $$  

(1)

The inversion for acoustic impedance is not linear; therefore, the above equation should be linearized by Taylor series approximation. This problem is an over-determined case, and thus the least square solution method can be used to solve it by minimizing the error vector (Cooke et al., 1983).
An initial geological model is necessary and defined from picked horizons and interpolated impedance values. The simple kriging of acoustic impedance well logs is used in this work. The simple kriging is a linear estimation geostatistical method which can mathematically be defined as follows (Kelkar et al., 2002):

$$X^* (\mathbf{u}_0) = \lambda_0 + \sum_{i=1}^{n} \lambda_i X (\mathbf{u}_i)$$

(2)

where, \(X(\mathbf{u}_i)\) denotes the value at neighboring location and \(X^* (\mathbf{u}_0)\) is the estimated value at unsampled location. \(\lambda_i\) is the weight assigned to the neighboring value points and depends on spatial relationship between unsampled and neighboring value points. Variogram is the common spatial relationship used in geostatistics. The value of variogram for a given lag distance \(L\) is the variance of pair data \(X (\mathbf{u}), X (\mathbf{u} + L)\). The half of variogram is described as the semi-variogram.

![Figure 6](image)

Relative AI obtained from SCI on K4 horizon used in horizontal spatial analysis.

Zones must be defined before creating initial model. A zone is an interval bounded by two seismic horizons. At the base or top of the model, the boundaries of a zone are one horizon and the base or top boundary of the model. In this study, three zones are introduced. Each zone is used as a stratigraphic interval for interpolating well data laterally to generate a 3D impedance model. Stratigraphic layering in each zone is proportional to horizon. For each zone, a separated variogram model (Table 1) and kriging system solution are used. The initial model described herein is shown in Figure 7. The initial model is created by using only log data, which have low horizontal resolution and thus the initial model has a low lateral resolution and model is smooth in lateral directions (Figure 7).

The results of variogram analysis are shown in Table 1. The vertical variogram analysis proposes an average variogram model with a range of 22 milliseconds and a sill value of 1; these results are also used in geostatistical inversion process.
The vertical ranges of variograms are in milliseconds, while the horizontal ranges of variograms are specified in meter. Therefore, a conversion of units must be applied to the range of variograms. After that anisotropy ratio is computed in each zone. The anisotropy ratio of vertical to horizontal direction is 1 to 13 up to 1 to 24 in different zones. This ratio in horizontal planes is 1 to 1.14 on average. The sills of variograms are assumed to a normalized value of one. The variogram models in three dimensions have been specified as anisotropic spatial functions by changing the ranges of the models. The vertical direction (z axis) of the models is parallel to the time axis. However, the horizontal directions (x and y axis) are parallel to the bedding. The azimuth of the x axis is 42° and three dimensional anisotropic variogram models are generated corresponding to this orientation. Further information on modeling anisotropy in 3D is referred to Deutsch and Journel (1992).

**Table 1**

<table>
<thead>
<tr>
<th>Zone (inline/cross-line)</th>
<th>Sill (values normalized between 0-1)</th>
<th>Range (meters)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1 (inline)</td>
<td>0.85</td>
<td>900</td>
<td>Exponential</td>
</tr>
<tr>
<td>Zone 1 (cross-line)</td>
<td>0.92</td>
<td>1000</td>
<td>Exponential</td>
</tr>
<tr>
<td>Zone 2 (inline)</td>
<td>0.87</td>
<td>1400</td>
<td>Exponential</td>
</tr>
<tr>
<td>Zone 2 (cross-line)</td>
<td>1</td>
<td>1200</td>
<td>Exponential</td>
</tr>
<tr>
<td>Zone 3 (inline)</td>
<td>0.90</td>
<td>1800</td>
<td>Exponential</td>
</tr>
<tr>
<td>Zone 3 (cross-line)</td>
<td>0.98</td>
<td>1600</td>
<td>Exponential</td>
</tr>
</tbody>
</table>

**Figure 7**

AI model is created by simple kriging of AI log data.

The initial model and wavelet is applied to deterministic inversion in the GLI process. The final result of deterministic inversion is illustrated in Figure 8 at a well location in impedance domain. Residual section, which is the difference between real and synthetic seismic trace, is shown in Figure 9. The
residual is useful as a simple and easy method for checking the quality of inversion results. The root mean square (RMS) of the amplitude of the residual section compared to the RMS of the inversion results is lower than 0.1 in different seismic lines; this is a perfect result for inversion process.

Figure 8
Quality check between a well of the studied field and deterministic seismic inversion results in acoustic impedance domain.

Figure 9
Residual seismic section; it illustrates dissimilarity between synthetic deterministic inversion and real seismic section.
5. Geostatistical seismic inversion

Geostatistical inversion based on spectral simulation (and compared with deterministic inversion) is graphically explained in the flow chart of Figure 10. GSI simulates low and high frequency information in a geostatistical framework by using well and seismic data together. The inversion is performed in the spectral simulation scene conditional on the well impedance data. The spectral simulation is based on the relationship among space property, frequency counterparts, and phase spectra (Yao et al., 2004). The spectral simulation shares different spectral components to build a priori geological model using well and variogram model parameter. The missing frequency band component must be simulated within spectral method and added to the model. If the spatial variable of the field in frequency domain is known as \(X(\omega)\), then \(|X(\omega)|\) is an amplitude spectrum and the term of density spectrum is referred to \(|X(\omega)|^2\). The amplitude spectrum can be honored globally over the entire field instead of only within search neighboring as the sequential simulation methods do. A global density spectrum is computed from spatial correlation model once and inverse Fourier transform is performed just once to generate a realization. Therefore, this simulation method is fast and fast Fourier transform was also used herein to further increase the simulation speed. Before the spectral simulation is performed, the probability density function (PDF) of acoustic impedance values at the well location is transformed into a Gaussian PDF by a normal score transformation. In GSI, the input data is composed of seismic, impedance log, horizon data, and wavelet. These data are used to build the initial model as well as the deterministic inversion. Eventually, the input data, the initial model, and the deterministic inversion are fed as inputs to this algorithm. The standard deviation of the kriging associated to the initial model is computed and termed as error grid map. The error grid map is standardized to values on a zero-to-one scale. This map is useful to provide a spatial well constraint for the GSI process. The spatial constraints provide the seismic contribution conditioned to the wells. These constraints are strong when they are close to the well locations (error grid values are zero at well location) and weak when away from the well locations at distances greater than the radii of well influence (error grid increases to one). The radii of well influence are obtained by variogram analysis (variogram range). Therefore, at well locations, the well log data are important to build a GSI model, and away from the wells, the seismic contribution is increased. The geostatistical methods can provide posterior uncertainty analysis, which is useful for reservoir characterization process.

6. Results and discussion

The output data of the GSI are impedance realizations. The GSI is not unique in solution; each possible solution is referred to as a realization of the simulation. Three of twenty realizations of the GSI in this studied field and the mean of the all realizations are shown in Figure 11. The inversion results are validated by original log data around well location. The statistical analysis of deterministic and geostatistical inversion results and comparison with the original log data are given in Table 2. The mean and standard deviation of the results are similar to the log data. It shows that the seismic volume is well inverted in both methods. The inversion results are subtracted from the original log data and the obtained errors. The mean of the errors are prearranged in Table 2. The mean of the error of the GSI is lower than that of the deterministic method. Therefore, the GSI has a better performance than the deterministic method. The improvements of the results can be seen in Figure 12. These charts show that the inversion results are well correlated in both methods, while the GSI improves the predicted AI.
Figure 10
A graphical description of comparison between deterministic and geostatistical seismic inversion methods.

Figure 11
Three GSI realizations of 20 realizations in AI domain (a, b, and c) and their averages (d).

The seismic data are band-limited; therefore, the deterministic inversion method cannot recover the absolute values of the acoustic impedance from seismic trace directly. Seismic original data in this...
field cover a frequency band in the range of 10 to 70 Hz. A lower 10 Hz pass frequency filter is applied to both inversion results, namely deterministic and geostatistical (Figure 13). It is obvious that low frequency part is hidden in the deterministic results, and the resolution of the GSI method is higher than that of the deterministic inversion. Improving the resolution of the inversion model is useful for other steps in reservoir characterization process. Deterministic inversion result, as shown in Figure 12, is correlated with log data; it is also observed that absolute impedance values are not correctly recovered and the deterministic inversion result is smooth.

Table 2
Statistical analysis of the results in AI domain (m/s×g/cm³).

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Mean of error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original AI log data</td>
<td>14797</td>
<td>2055</td>
<td>0</td>
</tr>
<tr>
<td>Deterministic AI inversion</td>
<td>14101</td>
<td>2408</td>
<td>615</td>
</tr>
<tr>
<td>Geostatistical AI inversion</td>
<td>15052</td>
<td>2273</td>
<td>355</td>
</tr>
</tbody>
</table>

Figure 12
The AI results of geostatistical (a) and deterministic inversion (b) correlated with the original AI log.

Another advantage of the GSI method compared to the deterministic inversion method is the uncertainty associated with interpolation, which can be computed in the GSI, while it is ignored in the deterministic inversion process. A set of realizations of acoustic impedance is used to represent the
uncertainty in the seismic inversion. Figure 14 illustrates the standard deviation of all the realizations, which were previously simulated. Standard deviation values around the well location are low. Also, this analysis shows that variability is increased at some positions where the well data are slightly constrained. Hence the uncertainty in the GSI method is related to well constraints in a way that a minor constraint on well data increases the uncertainty of the final GSI model.

Figure 13
Low frequency pass filter is performed on the deterministic results (top) and the GSI method (bottom).

Figure 14
Standard deviation map of all the realizations in AI domain; variability is increased at the bottom model.
5. Conclusions

The current work presents a new approach to the inversion of 3D post-stack seismic data. This approach incorporates well log data with seismic data using a spectral simulation. This method is successfully applied to the gas carbonate field data and obtained some conclusions as follows:

1- Low pass frequency filter on the inversion results shows that the low frequency part is hidden in the deterministic results, while it can be recovered in the GSI method;

2- The GSI, compared to the deterministic inversion, improved the AI prediction;

3- The variability in the GSI realizations depends on well constraints.

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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>AI</td>
<td>Acoustic impedance</td>
</tr>
<tr>
<td>GLI</td>
<td>Generalized linear inversion</td>
</tr>
<tr>
<td>GSI</td>
<td>Geostatistical seismic inversion</td>
</tr>
<tr>
<td>( \tilde{L} )</td>
<td>Lag distance</td>
</tr>
<tr>
<td>( M )</td>
<td>Model parameter</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability density function</td>
</tr>
<tr>
<td>RMS</td>
<td>Root mean square</td>
</tr>
<tr>
<td>SGS</td>
<td>Sequential Gaussian simulation</td>
</tr>
<tr>
<td>( T )</td>
<td>Data vector</td>
</tr>
<tr>
<td>( X(\tilde{u}_i) )</td>
<td>Sample value at neighboring location</td>
</tr>
<tr>
<td>( X^*(\tilde{u}_0) )</td>
<td>Estimated value at unsampled location</td>
</tr>
<tr>
<td>( X(\omega) )</td>
<td>Spatial variable of field in frequency domain</td>
</tr>
<tr>
<td>(</td>
<td>X(\omega)</td>
</tr>
<tr>
<td>( \lambda_i )</td>
<td>Weight assigned to the neighboring value points</td>
</tr>
</tbody>
</table>

Reference


