

Characterization of Reservoir Heterogeneity by Capacitance-resistance Model in Water-flooding Projects

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Received: May 31, 2015; *revised:* August 12, 2015; *accepted:* October 26, 2015

Abstract

Tedious calculations and simulations are needed to obtain an efficient production scenario and/or proper field development strategy. Capacitance-resistance model (CRM) is proved to be a fast reservoir simulation tool using just the field-available data of production and injection rates. This approach sets a time-constant and a weighting factor (or well-pair connectivity parameter) between each pair of injection and production wells according to their histories. In this study, we investigated the behavior of the CRM parameters in synthetic reservoir models with different porosity and permeability maps. Four reservoirs are considered with different porosities and permeabilities to study their effects on CRM response. We defined a new parameter, named error to mean production ratio (EMPR), to analyze the CRM performance. Some fluctuations are exerted on the production data to evaluate the capability of CRM against variable production records. Porosity showed a stronger effect on CRM parameters than the permeability based on the calculated EMPR. Unstable production history would result in large error which can be corrected with some smoothing techniques on variable production data. Also, a linear trend of EMPR was obtained with the change of porosity and permeability or a combination of the two parameters within the reservoir.

Keywords: Capacitance-resistance-model (CRM), Water Flooding, History Match, Reservoir Heterogeneity, Well Connectivity

1. Introduction

Different scientists have tried to combine petrophysics, geophysics, and thermodynamics with economic factors in order to propose the best production scheme (Mamghaderi and Pourafshari, 2013). Present commercial simulators are complex to handle and time-consuming; therefore, having an overview by less primary data is necessary to manage the field and to optimize the recovery. It was a trigger for reservoir engineers to develop a fast and reliable simulator. Simple predictive models, which usually use material or energy balance on a reservoir to evaluate its performance, are very fast

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and cost effective. Such simple approaches could have a preliminary estimation of reservoir performance and its characteristics with just a minimum amount of reservoir data such as production and injection rates; furthermore, they have really less run-time compared to other simulation techniques (Sayarpour, 2008).

Capacitance-resistance-model (CRM) is a quick reservoir performance evaluator, which has been recently applied to simulate reservoirs. It, therefore, can be used to analyze reservoirs during flooding projects (Cao et al., 2014). In this model, input and output values are injection and production rates respectively. This model is based on the parameters of time-constant and connectivities between each pair of injection and production wells (Mamghaderi and Pourafshari, 2013). Unlike the grid-based numerical-simulators, CRM would simulate the reservoir according to these two parameters, which are measures of the reservoir permeability and well-pairs interactions (Sayarpour, 2008).

The background of CRM is started from Albertoni (2002) and Lake who first introduced the well connectivity concept based on the injection and production rates. Next, Gentil (2005) presented the physical meaning of well connectivity by considering bottom-hole pressure fluctuations. Mathematical CRM has first been developed by Yousef et al. (2006) by incorporating well-pair connectivities and time constants in an overall material balance. After that, semi-analytical solutions of CRM governing equations were obtained by Sayarpour et al. (2008) by using superposition in time based on different reservoir control volumes. Weber et al. (2007) drew the conclusion that CRM could deal with large data sets by reducing the model parameters. This model has been implemented for different field cases, including primary recovery, water flooding, and gas flooding (Lee et al., 2011; Wang et al., 2011; Parekh and Kabir, 2011; Salazar et al., 2012; Nguyen, 2012; Can and Kabir, 2012; Tafti et al., 2013; Soroush et al., 2014). Moreover, it has been used to detect faults in a reservoir by analyzing the CRM parameters during water flooding (Masoumi et al., 2013).

Three different control volumes could be developed by analytical solutions for the continuity equation according to superposition in time and space, including 1) CRMT, a solution based on the entire field volume, 2) CRMP, a solution based on the drainage volume of a producer, and 3) CRMIP, a solution based on the control volume between injector-producer pairs. These solutions have been obtained using a linear variation of projections of producers bottomhole pressures (BHP) and a stepwise or linear variation of injection rate. Furthermore, CRM-block analytical solutions can be developed based on superposition in time and space if a series of tanks between each injector-producer pair is considered (Sayarpour, 2008). In mature fields, finding out the injected water distribution holds some valuable merits to have a reliable reservoir analysis (Moreno, 2013). CRM's could match production history for the entire field, well groups, or individual well, and, consequently, they can predict and optimize a field, well groups, or a specific production well. Since CRM's only predict total liquid production, it is necessary that an oil fractional-flow model be used as a function of time for immiscible water-floods and miscible CO₂ floods. Regarding to these fractional-flow models, oil production for a single producer, a group of producers, or the entire field can be estimated (Sayarpour, 2008).

In the present work, four different reservoirs are considered with different porosities and permeabilities. To distinguish the most effective parameter, CRM results are evaluated carefully and the effect of each parameter is reported. Furthermore, the effect of production fluctuations is studied to analyze the CRM behavior. A smoothing technique is implemented to compensate the negative effects of variable production records.

2. CRM concept and formulation

First, it is important to derive the CRM fundamental equation, which needs a mass balance as given below:

$$\begin{aligned} &\{\text{Mass of } j \text{ in c. v at } t + \Delta t\} - \{\text{Mass of } j \text{ in c. v at } t\} \\ &= \{\text{Mass of } j \text{ entering during } \Delta t\} - \{\text{Mass of } j \text{ leaving during } \Delta t\} \end{aligned} \quad (1)$$

We know that:
$$\frac{V_P C_t}{\bar{B}_O} \frac{d\bar{P}}{dt} = -\tilde{q}_{OSC} \quad (2)$$

Then

$$V_P C_t \frac{d\bar{P}}{dt} = i(t) - q(t) \quad (3)$$

It is also known that:

$$q = J(\bar{P} - P_{wf}) \quad (4)$$

For the control volume of a producer, shown in Figure 1, and from the continuity equation, the CRMP governing differential equation, which represents in-situ volumetric balance over the producer effective pore volume, is developed.

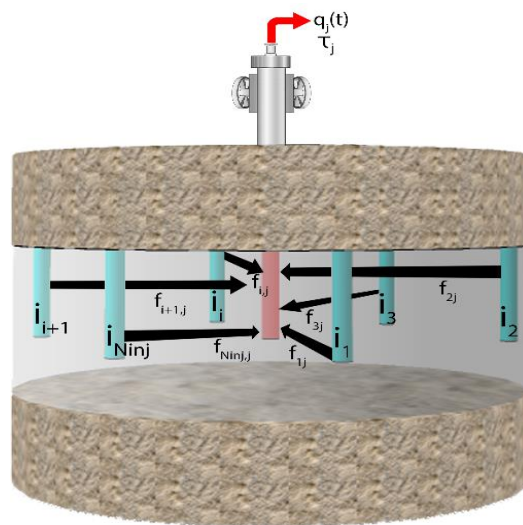


Figure 1

A schematic view of the production well, its drainage area, injection wells, and related CRM parameters.

Governing differential equation for this capacitance model is as follows (Liang et al., 2007):

$$\frac{dq_j(t)}{dt} + \frac{1}{\tau_j} q_j(t) = \frac{1}{\tau_j} \sum_{i=1}^{N_{inj}} f_{ij} i_i(t) - J_j \frac{dP_{wf,j}}{dt} \quad (5)$$

where, τ_j is time constant for producer j and is described as:

$$\tau_j = \left(\frac{C_t V_p}{J} \right)_j \quad (6)$$

The f_{ij} term, or well-pair connectivity, demonstrates the steady-state rate fraction of injector i , flowing towards producer j . The parameter of f_{ij} would be calculated as:

$$f_{ij} = \frac{q_{ij}(t)}{i_i(t)} \quad | \quad f_{ij} \geq 0 \quad , \quad \sum_{i=1}^{N_{inj}} f_{ij} \leq 1 \quad (7)$$

The sum of connectivities for any injectors should be used with caution, and they must be less than or equal to one; f_{ij} s must be positive values. These limiting constraints should be satisfied when CRMP parameters are evaluated. Solution for Equation 5 with BHP variations could be written as:

$$q(t) = q(t_0) e^{-\frac{(t-t_0)}{\tau_j}} + e^{-\frac{t}{\tau_j}} \int_{\varepsilon=t_0}^{\varepsilon=t} e^{\frac{\varepsilon}{\tau_j}} \frac{1}{\tau_j} \sum_{i=1}^{N_{inj}} f_{ij} i_i(\varepsilon) d\varepsilon - e^{-\frac{t}{\tau_j}} \int_{\varepsilon=t_0}^{\varepsilon=t} e^{\frac{\varepsilon}{\tau_j}} J_j \frac{dP_{wf,j}}{d\varepsilon} d\varepsilon \quad (8)$$

Integrating Equation 8 by parts would lead to the following equation:

$$q(t) = q(t_0) e^{-\frac{(t-t_0)}{\tau_j}} + \sum_{i=1}^{N_{inj}} \left[f_{ij} \left(i_i(t) - e^{-\frac{(t-t_0)}{\tau_j}} i_i(t_0) \right) \right] - e^{-\frac{t}{\tau_j}} \int_{\varepsilon=t_0}^{\varepsilon=t} e^{\frac{\varepsilon}{\tau_j}} \left(\sum_{i=1}^{N_{inj}} f_{ij} \frac{d i_i(\varepsilon)}{d\varepsilon} + J_j \frac{d P_{wf,j}}{d\varepsilon} \right) d\varepsilon \quad (9)$$

If the productivity index and injection rates for all injectors are considered to be a constant value during the time interval Δt_k , where $i_i(t) = I_i$, and a linear BHP drop is assumed for producer j from time t_0 to t , Equation 9 can be integrated as:

$$q(t) = q(t_0) e^{-\frac{(t-t_0)}{\tau_j}} + \left(1 - e^{-\frac{(t-t_0)}{\tau_j}} \right) \left[\sum_{i=1}^{N_{inj}} [f_{ij} I_i] - J_j \tau_j \frac{\Delta P_{wf,j}}{\Delta t} \right] \quad (10)$$

If production wells produce at constant BHP, the pressure term of Equation 10 then becomes zero.

Overall, the objective function would be the following equation.

$$obj_{fun} = \min \left(\sum_{k=1}^{n_t} \sum_{j=1}^{n_p} (q_{jk}^{obs} - q_{jk})^2 \right) \quad (11)$$

In Equation 11, q^{obs} is the observed production rate from production wells. The indexes j and k refer to production well number and time step respectively. n_p and n_t are the total number of production wells and time steps. By solving this equation, the optimum values of time constant and well-pair connectivities would be found for the history of a reservoir.

3. Methodology description

CRM could be used for near reservoir forecasting by adjusting two parameters of time constant and well-pair connectivity. It is of prime importance to know the effect of petrophysical parameters variations on CRM results. Undoubtedly, if reservoir engineers have a sense about CRM responses to different rock properties, they can develop more sustained production scenario.

In this work, four synthetic reservoir models are used to investigate the CRM response to petrophysical variations. A homogenous reservoir is considered as the base model with the dimensions of $15 \times 15 \times 15$ and a cell size of $150 \times 150 \times 50$ ft³. To develop three other reservoir models, the base model is divided into four different regions as shown in Figure 2, and variations in porosity, permeability, and their combination are exerted on the model. Regions are numbered the same as the injection well number within them. For instance, the first injection well is located in region one, and so on. Regions 1 to 3 have the dimensions of $5 \times 5 \times 15$. Common features of the reservoir models (rock and fluid) and properties of these regions are presented in Tables 1 and 2 respectively. Meanwhile, the porosity and permeability of the base model are 0.15 and 300 md respectively. In essence, only rock and fluid properties control the strength of well-pairs; therefore, fluid properties are also effective in CRM results. On the one hand, time constant is a function of total compressibility and productivity index, which are dependent on rock and fluid properties; on the other hand, different rock and fluid properties would result in different relative permeability functions, and, as a consequence, various well-pair connectivities would be yielded for different rock and fluid properties.

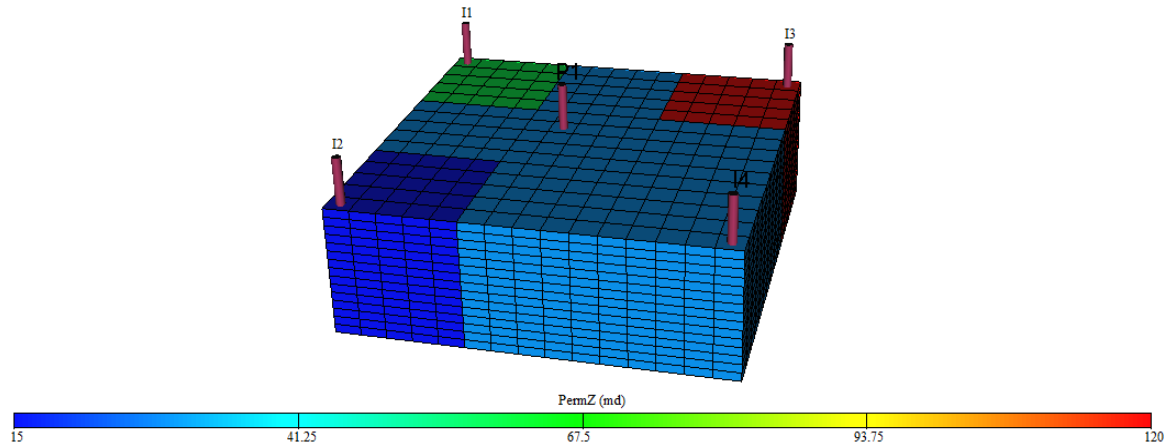


Figure 2

Reservoir schematic view, wells, and regions locations.

Table 1
Common features of all reservoir models.

Property	Value	Unit
BHP of the extractor	2500	psi
Initial reservoir pressure	3500	psi
Initial reservoir temperature	209	°F
Compressibility of matrix	1E-06	psi ⁻¹
Compressibility of water	1.1 E-06	psi ⁻¹
Oil density (dead oil)	53.76	lb./ft ³
Water density	62.43	lb./ft ³
Water viscosity (reference pressure = 4100 psi)	1.01	cP

Table 2
Porosities and permeabilities of different regions in four reservoir models.

Model	Porosities of different regions				Permeabilities of different regions			
	1	2	3	4	1	2	3	4
Base	ϕ	ϕ	ϕ	ϕ	k	k	k	k
Porosity zoned	2ϕ	0.5ϕ	4ϕ	ϕ	k	k	k	k
Permeability zoned	ϕ	ϕ	ϕ	ϕ	2k	0.5k	4k	k
Porosity-permeability zoned	2ϕ	0.5ϕ	4ϕ	ϕ	2k	0.5k	4k	k

Water flooding is performed for about 1000 days through four corner injectors and reservoir fluid is produced by one central production well. Injection profiles of four injectors and production rates are shown in Figures 3 and 4 respectively. In addition to the production and injection data, which are available from history, CRM simulator needs some initial guesses to predict the reliable parameters for reservoir forecasting. Initial conditions, which are used for all the simulations of this study, are 10, 1, and 300 for time constants, connectivities, and oil rates respectively. These initial conditions are not mandatory; however, it should be noticed that choosing initial conditions near the boundaries is not recommended since the model might fail during the optimization process. The term “near the boundaries” refers to conditions which variables could not search all their allowed criteria to find the optimum value.

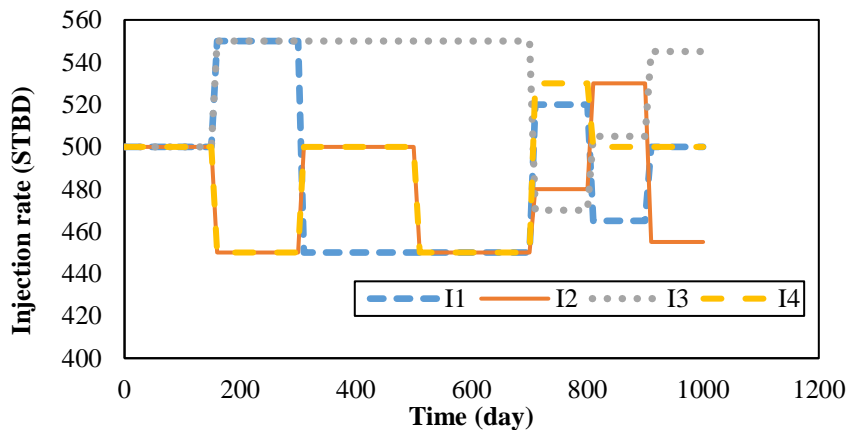
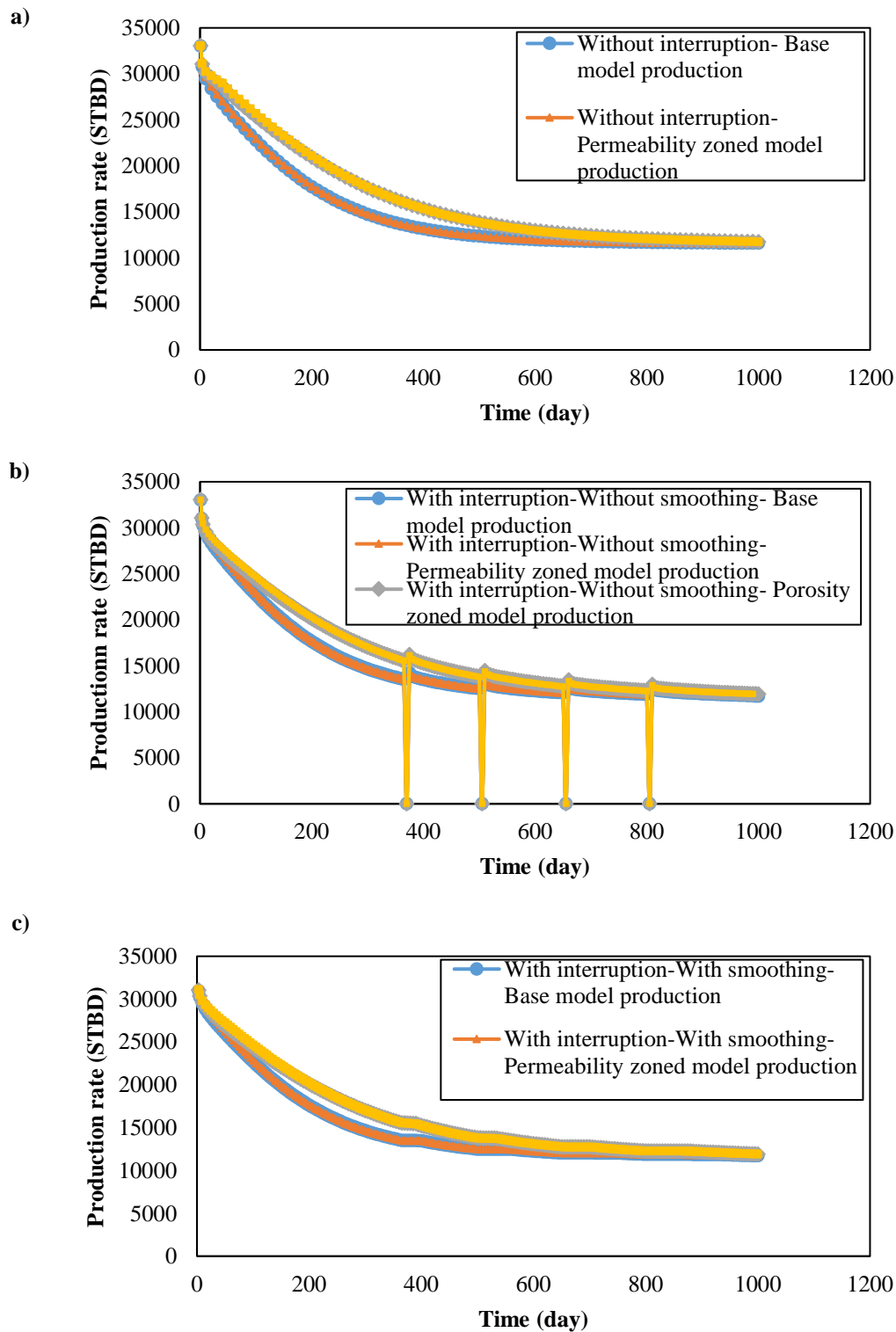


Figure 3
Injection rate pattern for four injection wells.

**Figure 4**

Production rates, used as CRM inputs; a) without interruption, b) with interruption and without smoothing, c) with interruption and with smoothing.

Three different scenarios are considered to study the petrophysical properties variations. These scenarios are 1) studying four reservoir models without production fluctuations, 2) studying four reservoir models with production fluctuations and without smoothing, and 3) studying four reservoir

models with smoothed production profile. The process behind smoothing is as follows:

If a production well has been shut-in and opened after sometimes, its production rate would be more than the rate just before the shut-in until several time-steps. Considering the following equation:

$$\text{Scaling factor} = \frac{q_b - q_a}{n + 1} \quad (12)$$

where, q_b is the production rate just before shut-in, and q_a is the first production rate after the shut-in period, which is lower than q_b ; n is the number of time steps between q_b and q_a . Now by subtracting the scaling factor from q_b , a new production rate would be calculated for the next time step. Continuing this algorithm for $n+1$ time steps would result in a new production data set, which has no fluctuation; moreover, cumulative production by this algorithm is the same as the model with fluctuations. The schematic view of production data sets for all scenarios is shown in Figure 5.

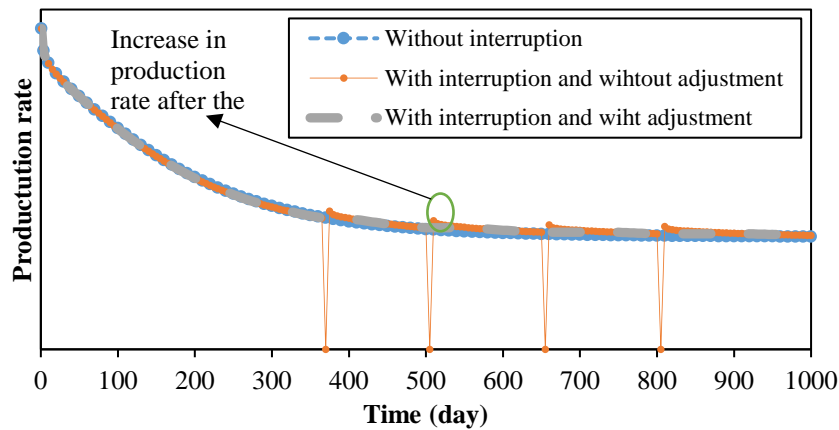


Figure 5

A schematic view of three production scenarios (time of interruptions are the same for all the models).

To find out the performance of CRM just with a quick glance at its results, three parameters of MPR, EMPR, and RMPR are defined and used. These three parameters stand for mean-production-rate, error to MPR ratio, and real-mean-production-rate of reservoir respectively. MPR and EMPR are calculated based on the CRM results and RMPR is determined from historical record of the reservoir. Mathematical forms of these parameters are presented below:

$$MPR = \frac{\text{Sum of production rates, calculated by CRM at each time step}}{\text{number of time steps}} \quad (13)$$

$$EMPR = \frac{\text{Error of CRM calculations}}{MPR} \quad (14)$$

$$RMPR = \frac{\text{Sum of production rates, reported in reservoir history at each time step}}{\text{number of time steps}} \quad (15)$$

4. CRM simulation results and discussion

The CRM simulation results for all the considered cases are presented in Tables 3 and 4. As shown in Table 3, for the production data with no fluctuations, the models with porosity and permeability

changes show a low error, while the homogeneous base model has the highest error. It should be noted that the order of increase in permeability and porosity is the same for all the models; however, porosity affected the CRM parameters more than permeability. The models are ranked in the following order in terms of their error (when no production fluctuation exists): “the model that has four regions with different porosity and permeability values < the model that has four regions with different porosity values < the model that has four regions with different permeability values < the base model”. This order is the same when the models have smoothed production records.

Table 3
CRM results for different reservoir models.

CRM results for reservoir models without production interruption					
Model	τ	Error (RB/D)	MPR (RB/D)	EMPR	RMPR (RB/D)
Base	224.49	810	14691	0.055	15048
Porosity zoned	299.24	532	16340	0.033	16450
Permeability zoned	221.85	778	14703	0.053	15056
Permeability-porosity zoned	296.02	512	16455	0.031	16500
CRM results for reservoir models with production interruption					
Model	τ	Error (RB/D)	MPR (RB/D)	EMPR	RMPR (RB/D)
Base	46.34	6134	9785	0.627	14870
Porosity zoned	57.92	7337	9813	0.748	16010
Permeability zoned	48.90	6140	9787	0.627	14877
Permeability-porosity zoned	64.78	7369	9835	0.749	16058
Adjusted CRM results for reservoir models with production interruption					
Model	τ	Error (RB/D)	MPR (RB/D)	EMPR	RMPR (RB/D)
Base	236.69	936	14661	0.064	14870
Porosity zoned	300	671	15937	0.042	16010
Permeability zoned	233.29	921	14669	0.063	14877
Permeability-porosity zoned	300	657	16016	0.041	16058

Table 4
Well-pair connectivities.

Well-pair connectivities for reservoir models without production interruption				
Model	$f1j$	$f2j$	$f3j$	$f4j$
Base	1	1	1	1
Porosity zoned	0.99	1	0.98	1
Permeability zoned	1	0.87	1	1
Permeability-porosity zoned	1	0.95	1	1

Well-pair connectivities for reservoir models with production interruption				
Model	f_{1j}	f_{2j}	f_{3j}	f_{4j}
Base	0.72	0.77	0.73	0.74
Porosity zoned	0.66	0.69	0.65	0.68
Permeability zoned	0.56	0.50	0.57	0.55
Permeability-porosity zoned	0.62	0.57	0.61	0.62
Adjusted Well-pair connectivities for reservoir models with production interruption				
Model	f_{1j}	f_{2j}	f_{3j}	f_{4j}
Base	0.87	0.85	0.90	0.89
Porosity zoned	0.83	0.86	0.83	0.85
Permeability zoned	0.77	0.72	0.79	0.76
Permeability-porosity zoned	0.79	0.76	0.78	0.80

Models with production fluctuation could not be interpreted readily since they have high error values. An increase in porosity means that the initial oil in place has increased, which leads to higher capacitance and time constant of the model. Based on the concept of EMPR and with the same amount of injection, the more accurate model has a lower EMPR since it has a lower error and higher production. Therefore, the order of EMPR for the models without production fluctuation or with smoothed fluctuations is exactly the same. Furthermore, EMPR values show linear relationship with petrophysical properties variations if production data do not have fluctuations or have smoothed production records. For example, EMPR values would decrease respectively to 0.22 and 0.02 with variation in porosity and permeability, and EMPR would decrease to 0.24 if both of them are changed. Figure 6 shows this relationship with a bar chart.

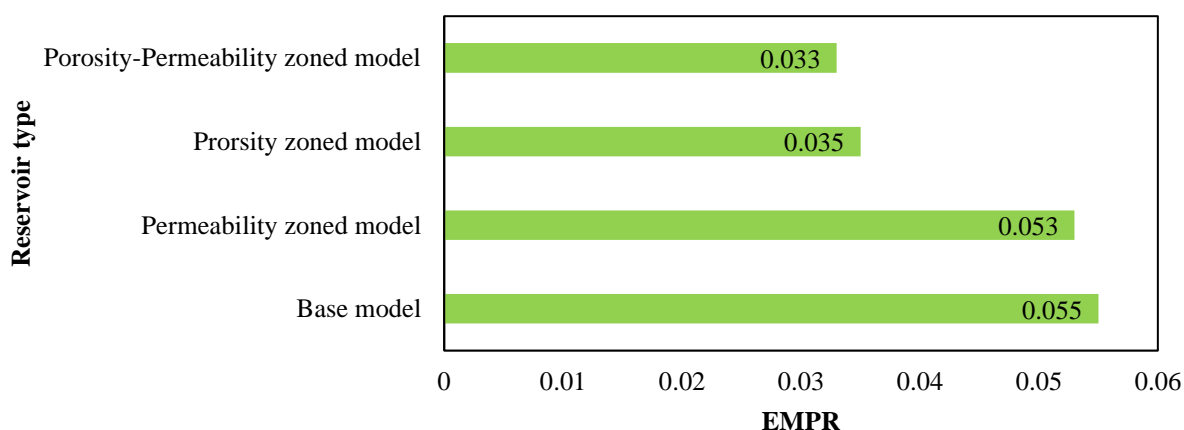


Figure 6

EMPR for different simulated models with CRM.

Table 3 also demonstrates that MPR would go to RMPR if fluctuated production data are smoothed. It should be mentioned that porosity change has a greater impact on EMPR compared to that of the permeability variations. These observations can be explained by Equation 10. The first term of the

right hand side of this equation relates to the primary depletion of the reservoir; therefore, an increase in porosity would increase this term remarkably, which results in a greater impact on EMPR compared to that of the permeability. Table 4 shows well-pair connectivities between each of the injection wells and different production wells. According to this table, well-pair connectivities are unity when the model is homogenous. By increasing the extent of heterogeneity through the reservoir, the values of f_{ij} would decrease from unity.

5. Conclusions

Different CRM simulations have been conducted with different reservoir rock properties to investigate their effects on CRM performance. It is shown that porosity changes would result in a greater impact on CRM results compared to permeability variations. Moreover, EMPR is much more sensitive to porosity variations than permeability variations. This parameter shows linear relationship with changing porosity and permeability. This might help engineers to better predict CRM performance when porosity and permeability are planned to change; examples are acidizing or hydraulic fracturing processes. Fluctuation in production data would result in a noticeable error in CRM results, and a modification is needed to compensate the fluctuation effect. It seems that CRM would not be a reliable simulation technique if the input production history has many fluctuations and some smoothing methodologies similar to what we proposed herein are necessary for better results.

Nomenclature

Variables	
c_t	: Total compressibility, [lt ² /m]
$c.v.$: Control volume
$EMPR$: Error to mean production rate, [--]
f_{ij}	: Interwell connectivity constant between injector/producer well pair, [--]
J	: Well productivity index, [l ⁴ /m]
MPR	: Mean production rate, [l ³ /t]
N	: Number of wells
n_p	: Total number of production wells
n_t	: Total number of time steps
\bar{P}	: Average reservoir pressure, [m/ lt ²]
P_{wf}	: BHP of the producer, [m/ lt ²]
q	: Total fluid rate, [reservoir l ³ /t]
q_a	: First production rate after the shut-in period that is lower than q_b , [l ³ /t]
q_b	: Production rate just before shut-in, [l ³ /t]
$q(t_0)$: Effect of production prior to the analysis period, [l ³ /t]
$RMPR$: Real mean production rate, [l ³ /t]
t	: Time, [t]
V_p	: Pore volume, [l ³]
Greeks	
ε	: Variable of integration
Δ	: Change in a certain quantity
τ	: Time constant, [t]
Subscripts	
i	: Injector index
j	: Producer index

k	: Time step index
inj	: Injector index
Superscripts	
obs	: Observed parameter

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