The Porosity Prediction of One of Iran South Oil Field Carbonate Reservoirs Using Support Vector Regression

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Abstract

Porosity is considered as an important petrophysical parameter in characterizing reservoirs, calculating in-situ oil reserves, and production evaluation. Nowadays, using intelligent techniques has become a popular method for porosity estimation. Support vector machine (SVM) a new intelligent method with a great generalization potential of modeling non-linear relationships has been introduced for both regression (support vector regression (SVR)) and classification (support vector classification (SVC)) problems. In the current study, to estimate the porosity of a carbonate reservoir in one of Iran south oil fields from well log data, the SVR model is firstly constructed; then the performance achieved is compared to that of an artificial neural network (ANN) model with a multilayer perceptron (MLP) architecture as a well-known method to account for the reliability of SVR or the possible improvement made by SVR over ANN models. The results of this study show that by considering correlation coefficient and some statistical errors the performance of the SVR model slightly improves the ANN porosity predictions.

Keywords: Petrophysical Parameter, Reservoirs, Porosity, Well Log Data, Support Vector Machine

1. Introduction

Porosity is a static property of reservoir rock used for characterizing the reservoirs. Although the porosity measured from the core sample is generally more acceptable than the log-derived porosity, there are some limitations for this measurement approach. Coring is costly and in general is performed only in few wells. Even in the cored wells, it may not include some intervals of the reservoir. In contrast, well log data provide continuous inexpensive measurements compared to the core sampling system for almost all exploratory wells. Some empirical formulas and multiple linear regression methods have been proposed to relate well log data and porosity, but they are all case dependent and cannot be extended to unknown reservoirs, especially of carbonate type. To predict the porosity in carbonate reservoir rocks, known to be highly heterogeneous, a highly non-linear regression model is required. Artificial neural network (ANN) a powerful intelligent method has successfully been applied to the determination of petrophysical parameters such as porosity and permeability from well log data (Mohaghegh et al., 1996; Helle and Ursin, 2001; Rezaee et al., 2008). There are yet some challenges to provide reliable and more accurate methods for porosity prediction. Recently, a new intelligent method called support vector machine (SVM) has been proposed with the

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ability of constructing both the classification and regression models (Vapnik, 1998). The intelligent method SVM is based on statistical learning approach. Unlike the conventional neural networks, which are based on the traditional empirical risk minimization (ERM) principle, SVM’s employ the structural risk minimization (SRM) principle. The difference between the ERM and SRM is that ERM minimizes the error on the training data, but SRM minimizes an upper bound on the expected risk. It has been shown that the SRM is superior to the ERM; thus it causes the SVM to have a greater ability to generalize, i.e. high performance in the unseen data. Generalization is the goal in any statistical learning (Gunn, 1998). In addition, for a given training set, the SVM approaches to the global minima in contrast to the traditional neural networks (Basak et al., 2007). The SVM method has successfully been applied to text detection (Kim et al., 2001), handwriting recognition (Choisy and Belaid, 2001), the prediction of time series (Drucker et al., 1997), and porosity estimation in a heterogeneous sandstone reservoir (Al-Anazi and Gates, 2010). In this paper, the porosity of a heterogeneous carbonate reservoir in an oil field located at the south of Iran is estimated using two different approaches. To determine the porosity of the reservoir, the ANN as a popular and powerful method and the SVR as a more recent and yet powerful intelligent method are used to construct the models. Firstly, the sensitivity of SVR model parameters is investigated and an optimum set of parameters are selected. Then, the performance of both the SVR and ANN models are assessed and compared.

2. Support vector machine

Support vector machine (SVM) method has been applied to both the classification and regression problems for which support vector classification (SVC) and support vector regression (SVR) terminologies are respectively used (Gunn, 1998). A simple case of SVC is a linearly-separable classification in which a linear classifier (a hyperplane) is looked for to perform the separation of some data examples required to be separated into two classes. The problem is that there are many hyperplanes which separate the two classes, called separating hyperplanes (Figure 1.a). A term margin is defined as the summation of the two shortest distances from the separating hyperplane to the nearest sample of each class, i.e. the distance between H1 and H2 in Figure 1.b. By SVM method, when the margin is maximized, the optimal separating hyperplane is achieved.

![Figure 1](image)

Classification of positive and negative samples by (a) separating hyperplanes and (b) optimal separating hyperplane H

2.1. Support vector regression

Support vector machine is developed for regression problems by the introduction of the ε-insensitive loss function (Vapnik, 1995) given by:

\[
|f(x) - y|_\varepsilon = \begin{cases} 
0 & \text{for } |f(x) - y| \leq \varepsilon \\
|f(x) - y| - \varepsilon & \text{otherwise}
\end{cases}
\] (1)
where, \( y \) is the target value. In linear regression, a linear function \( f(x) \) is described as:

\[
f(x) = w \cdot x + b
\]  

(2)

where, \( w \) is the weight vector; \( b \) is bias and \( \cdot \) denotes the dot product. In epsilon-support vector regression (\( \varepsilon \)-SVR), a function \( f(x) \) is searched so that it is at the maximum \( \varepsilon \) deviation from the target value \( y \), and simultaneously ensures the flatness. The flatness in Equation 2 is satisfied by minimizing the norm of \( \|w\|^2 \). Introducing the slack variables \( \xi_i \) and \( \xi_i^* \) to be the upper and lower constrains on the function \( f(x) \), the optimal regression function is given by:

\[
\text{Minimize} \quad \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{l} \xi_i + \xi_i^*
\]  

(3)

with constrains:

\[
\begin{aligned}
&y_i - w \cdot x - b \leq \xi_i + \varepsilon \\
w \cdot x + b - y_i \leq \xi_i^* + \varepsilon \\
&\xi_i, \xi_i^* \geq 0
\end{aligned}
\]  

(4)

where, \( C \) is a positive parameter determining the trade-off between minimizing the flatness of the function \( f(x) \) and the amount up to which deviations larger than \( \varepsilon \) are tolerated (Smola, 2004). The graphical view to the solution of linear SVR is given in Figure 2. The points outside the shaded region are called support vectors and are the sufficient points to construct the model. The points inside the region are ignored and can be removed (Smola, 2004; Üstün et al., 2006).

By Lagrangian formulation, Equation 3 with constrains given in Equation 4 can be solved and the linear regression reads:

\[
f(x) = \sum_{i=1}^{l} (\alpha_i - \alpha_i^*) x_i \cdot x + b
\]  

(5)

where, \( \alpha_i \) and \( \alpha_i^* \) are Lagrange multipliers. This linear regression is now ready to be tested on any unseen data \( x \) (Basak et al., 2007).

2.2. Non-linear regression

The linear regression discussed is the basis of performing non-linear regression. In a non-linear regression, the input space is mapped by a mapping \( \varphi \) into a high dimensional space called the feature space. The mathematical formulations used for the linear regression are applied into the feature space. The mathematical formulations for the linear regression at the feature space are equivalent to those of the non-linear regression at the input space (Basak et al., 2007).
Since the feature space is in high dimensions, the solution in this space requires costly computations. Therefore, instead, a kernel function $K(x_i, x_j)$ is introduced in the input space to be equivalent to the dot product $\phi(x_i) \cdot \phi(x_j)$ in the feature space to address the course of dimensionality (Smola, 2004; Gunn, 1998). Hence the non-linear regression function is given by:

$$f(x) = \sum_{i=1}^{l} (a_i - a'_i) K(x_i, x) + b$$  \hspace{1cm} (6)$$

An example of the kernel functions is the Gaussian radial basis function (Gaussian RBF) kernel which is of the form of $K(x_i, x_j) = \exp(-\|x_i - x_j\|^2 / 2\sigma^2)$, in which $\sigma$ is the kernel parameter.

3. Methodology

The case study of this paper is a carbonate reservoir in an oil field located at the south of Iran. 1463 data series achieved from six wells of the reservoir containing the core-measured porosity and log data. The available well logs are sonic log (DT), bulk density log (RHOB), neutron log (NPHI), gamma ray log (GR), and two resistivity logs including deep laterolog (LLD) and shallow laterolog (LLS).

Some statistic information of the core porosity is provided in Table 1. The distribution of core porosity is also presented in Figure 3. In Figure 3, the core porosity (Core-PHI) distribution seems to be bimodal with the modes around 2% and 15%, but it totally has a mean of 8.5%. According to the correlation coefficient $R^2$, the relative importance of the log data and depth of the data used to predict the core porosity is presented in Figure 4, which shows that the most important log for prediction is the sonic log (DT) and then the neutron log (NPHI).

<table>
<thead>
<tr>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>Sample Variance</th>
<th>Kurtosis</th>
<th>Skewness</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0852</td>
<td>0.0751</td>
<td>0.0638</td>
<td>0.0041</td>
<td>-0.8366</td>
<td>0.4874</td>
<td>0.298</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Table 1
Descriptive statistics of core porosity data

Both of them together with the density log (RHOB) are known as the porosity logs used for porosity determination in empirical formulas. In this study, the logarithms of LLD and LLS are preferred to be used instead of LLD and LLS because the logarithms of LLD and LLS have greater correlations with porosity than LLD and LLS (see Figure 4). The logarithm of LLD and LLS are also more matched
with the normal distribution than LLD and LLS. The distributions of LLD and Log(LLD) are illustrated in Figure 5.

Figure 4
Relative importance of input variables for estimating the core porosity

Figure 5
The distribution of (a) LLD and (b) logarithm of LLD with normal distribution
To remove redundancy or duplication from the set of correlated variables, some methods such as factor analysis can be used. Herein, because the number of input variables is not too large, the best variables are selected by considering different numbers of input variables and checking the performance obtained. This is considered especially for DT, RHOB, and NPHI logs, in which great cross-correlation exists. The best performance is found to be using all of the seven variables, namely DT, RHOB, NPHI, Log(LLD), Log(LLS), GR, and Depth. However, it should be noticed that using all of the input variables does not necessarily improve the performance.

The general methodology for estimating core porosity from the log data in this study is that firstly all of the data are normalized and 20% of them are randomly assigned as the test set. The remaining data are used as the training and validation sets in a \( k \)-fold cross-validation process and to obtain the optimal parameters of the ANN and SVR models. The ANN and SVR models constructed should be then tested on the test set to compare the accuracy of each model. The accuracy measures used here are the correlation coefficient \( R \), the root mean square error (RMSE), the average absolute error (AAE), and the maximum absolute error (MAE). The mathematical expressions of each accuracy measure are given in Table 2.

### Table 2

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation Coefficient</td>
<td>( R = \frac{\sum_{i=1}^{l}(y_i - \bar{y})(\hat{y}<em>i - \bar{\hat{y}})}{\sqrt{\sum</em>{i=1}^{l}(y_i - \bar{y})^2 \sum_{i=1}^{N_p}(\hat{y}_i - \bar{\hat{y}})^2}} )</td>
</tr>
<tr>
<td>Root Mean Square Error</td>
<td>( \text{RMSE} = \sqrt{\frac{1}{l} \sum_{i=1}^{l}(y_i - \hat{y}_i)^2} )</td>
</tr>
<tr>
<td>Average Absolute Error</td>
<td>( \text{AAE} = \frac{1}{l} \sum_{i=1}^{l}</td>
</tr>
<tr>
<td>Maximum Absolute Error</td>
<td>( \text{MAE} = \max</td>
</tr>
</tbody>
</table>

### 3.1. ANN approach

To construct an ANN, the type of neural network model and the related parameters such as the number of hidden layers, the number of neurons in the hidden layers, etc. should be optimized or the typical values of them should be utilized. In this paper, the most commonly used architecture of ANN, the so-called multilayered perceptron, is used. The training algorithm of the model is the backpropagation (Ligtenberg and Wansink, 2001). In order to speed up the training, the Levenberg-Marquardt algorithm is used (Hagan et al., 1996). The number of hidden layers is the one which is the most common. The neurons into the input layer are the well logs and the depth, and the output layer consists of one neuron, namely the scalar core porosity, which should be predicted. A tangent sigmoid activation function is used for the hidden and output layers and the optimal number of neurons is obtained through a 10-fold cross-validation. For this study, the neural network toolbox in MATLAB environment (Matlab User’s Guide, 2010) is used.

### 3.2. SVR approach

To construct a model by SVR, firstly, the kernel function, and then the parameters of the model and
kernel function should be specified. Table 3 presents the kernel functions used in this study and their mathematical expressions. These kernels are three of the most commonly used kernels (Üstün et al., 2006) with up to one kernel parameter. The linear kernel is simple and does not have any kernel parameter; the kernel parameters of the polynomial and Gaussian RBF kernels are the degree (d) and width (σ) respectively. It should be noted that using any kernel function, SVR models consist of two additional parameters, namely the regularization constant (C) and the ε range, if the ε-insensitive loss function is used. Different values of the parameters may have a considerable effect on the accuracy achieved by the SVR. In order to obtain good accuracy by the SVR model, the optimal values of the model and kernel parameters should be used for constructing a proper model. These parameters can be obtained from the prior knowledge on the particular application under study. For instance, if previous works show that a polynomial kernel with the degree of 2 is a proper choice for an application, this can be used for further studies in that specific application. But without the prior knowledge, some optimizations should be performed (Gunn, 1998; Saffarrzadeh and Shadizadeh, 2012). In this study, for predicting porosity, a grid search method and 5-fold cross-validation is used to optimize the parameters.

### Table 3

Common kernel functions with up to one kernel parameter

<table>
<thead>
<tr>
<th>Kernel Function</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>( K(x_i, x_j) = x_i \cdot x_j )</td>
</tr>
<tr>
<td>Polynomial</td>
<td>( K(x_i, x_j) = (x_i \cdot x_j + 1)^d )</td>
</tr>
<tr>
<td>Gaussian RBF</td>
<td>( K(x_i, x_j) = \exp\left(-\frac{|x_i - x_j|^2}{2\sigma^2}\right) )</td>
</tr>
</tbody>
</table>

The advantage of the grid search is its ability to provide the global minimum (or the optimal point). However, it takes a long time for this method to obtain the optimal point, especially when a cross validation is also used (Al-Anazi and Gates, 2010). Therefore, in this paper, the required samples for each parameter are first reduced to decrease the time. After the global minimum is obtained, another grid search is performed on the points around the global minimum to refine the global point. To train and test the SVR models, the SVM-KM Matlab toolbox (Canu et al., 2005) is used. However, for SVM approaches, many different tools such as LIBSVM, SVMlight, SVMTorch, and DTREG can be used. Al-Anazi and Gates predicted porosity in a heterogeneous sandstone reservoir using DTREG software (Al-Anazi and Gates, 2010).

4. Results and discussion

To construct an SVR model, the SVR parameters should be optimized. Herein, the effect of these parameters on the accuracy is first considered, and then SVR and ANN models are constructed and compared.

4.1. Investigating the sensitivity of SVR parameters

The accuracy of SVR-derived porosity is highly affected by variation in the parameters of the SVR model and kernel function. Here, the sensitivity of accuracy with respect to the SVR parameters is considered when a Gaussian RBF is used as the kernel function. The parameters that should be optimized are C and ε, which are the model parameters, and σ which is the kernel parameter. In order to obviously investigate the effect of each parameter, the parameter under study is permitted to vary...
while the other parameters are set constant. Then, the RMSE’s of the 5-fold cross-validation of the normalized data are obtained and plotted versus that parameter. Firstly, the effect of parameter C on the accuracy is given. Figure 6 presents the RMSE’s of the cross validation with respect to the natural logarithm of C. In Figure 6, parameter C varies between 1 and 3000 and the global minimum of RMSE occurs at C=exp(5)=148.4, when the parameters ε and σ are constant and equal to 0.1 and 0.5 respectively. If the constant values of 0.001 and 0.4 are assigned to the parameters ε and σ respectively, the graph is altered and the global minimum of RMSE occurs at C=exp(3.5)=33.12.

Figure 6
Effect of C parameter on the accuracy of estimation while keeping ε and σ parameters constant

To analyze the effect of ε parameter on accuracy, the parameters C and σ are assigned constant values of 30 and 0.4 respectively and the cross-validation RMSE’s are plotted versus the natural log of ε (Figure 7.a). In Figure 7.a, the RMSE is highly affected by the high values of ε. However, almost the same RMSE’s are achieved at small values of ε. The question is that which of the ε values is suitable to be selected when almost the same RMSE’s are obtained. The small values of ε increase the runtime of the model. The effect of variation in ε on the relative time of cross-validation is shown in Figure 7.b.

Figure 7
Effect of ε parameter (a) on the accuracy and (b) on the relative time of cross-validation; the parameters C and σ are set at 30 and 0.4 respectively.
It represents that by choosing $\varepsilon$ to be 0.05 (exp(-3)), the running time is 4.4 times faster than when $\varepsilon$ is 4.5e-05 (exp(-10)), at which the minimum of error is obtained; but an increase of 0.002 in RMSE should be accepted. In addition, choosing $\varepsilon$ equal to 0.0183 (exp(-4)) causes the runtime to be almost 2 times shorter and to accept only a little increase of 0.0004 in the RMSE. This can be noticed when the problem of time limitation exists.

Figure 8 shows the effect of $\sigma$ parameter, varying from 0.03 to 20, while parameters $C$ and $\varepsilon$ are set at 30 and 0.001 respectively. The global minimum of RMSE occurs at $\sigma=\exp(-1.5)=0.22$ and very small values of $\sigma$ have a stronger effect on the accuracy than the larger values.

So far, the effect of variation in each parameter on the RMSE, while the other parameters are kept constant, has been investigated. The global minimum point achieved in each case is, in fact, the one achieved in the direction of the variable parameter and is altered when the constant parameters are changed. To find the global minimum point while all of the three parameters $C$, $\varepsilon$, and $\sigma$ are varying, an optimization method such as grid search is required. In this paper, to perform a grid search method, the search range of each parameter is specified and sampled into the required samples. Considering all of the SVR parameters, this can be viewed as a grid at any intersection ($C$, $\varepsilon$, and $\sigma$) points at which the SVR model is constructed by the training data and tested by the test data. The point providing the minimum of the mean square error (MSE) on the test data is referred to as the optimal point which gives the best values of the parameters to construct the proper model. Because in this study, a 5-fold cross-validation is used during the grid search process, the point giving the minimum of the average MSE on the validation set obtained by the 5-fold cross-validation is the optimal point. This method is also utilized to obtain the SVR parameters when linear or polynomial kernels are used.

4.2. Porosity prediction using SVR and MLP approaches

In this paper, for the MLP approach, the best value of the number of neurons in the hidden layer was obtained from a 10-fold cross-validation by checking over many different numbers of neurons. For the SVR approach, the best SVR parameters were obtained by a 5-fold cross-validation and grid search method as mentioned before. Then, the MLP and SVR models were constructed by the best values. The accuracy of each model should now be tested by the test dataset which is unseen to the models. The statistical errors and correlation coefficient $R$ of the test dataset for porosity estimation by ANN and SVR methods are presented in Table 4, in which the best accuracy is bolded. Among the three
kernel functions used in SVR model, the linear kernel provides better MAE. The statistical errors RMSE and AAE in Gaussian RBF kernel are smaller than those of other kernel functions used here. The performance of ANN has been improved by SVR models using the RBF and polynomial kernels because all of the statistical errors are smaller than those of the MLP. However, the improvement is not a lot. Using a linear kernel improves only the MAE of the ANN.

<table>
<thead>
<tr>
<th>Regression Method</th>
<th>RMSE</th>
<th>AAE</th>
<th>MAE</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLP (ANN)</td>
<td>0.0340</td>
<td>0.0238</td>
<td>0.1644</td>
<td>0.8510</td>
</tr>
<tr>
<td>Linear (SVR)</td>
<td>0.0352</td>
<td>0.0257</td>
<td>0.1390</td>
<td>0.8347</td>
</tr>
<tr>
<td>Polynomial (SVR)</td>
<td>0.0322</td>
<td>0.0232</td>
<td>0.1489</td>
<td>0.8648</td>
</tr>
<tr>
<td>RBF (SVR)</td>
<td>0.0293</td>
<td>0.0208</td>
<td>0.1484</td>
<td>0.8891</td>
</tr>
</tbody>
</table>

Comparing the correlation coefficients, SVR models using Gaussian RBF and polynomial kernels provide slightly better correlation coefficients than ANN model. The scatter plot of the predicted porosity versus core porosity is illustrated in Figure 9; it can be seen in this figure that slightly better porosity values are predicted by the SVR model compared to the ANN model.

5. Conclusions
In this paper, the effect of different SVR model parameters and Gaussian RBF kernel on the accuracy of porosity estimation and prediction was investigated. The porosity was then estimated using both SVR and ANN methods with an optimized set of parameters. Through measures of statistical errors and correlation coefficients, it was shown that the SVR models using the polynomial and Gaussian RBF kernel functions could slightly improve the performance of porosity estimation by ANN. Generally, it can be concluded that the Gaussian RBF kernel can provide slightly better results than the linear and polynomial kernels and ANN and is considered as a reliable and good alternative to the ANN method, especially when the size of training and test data are not large enough. Finally, it should be noted that another popular kernel function, other than the linear, polynomial, and Gaussian...
RBF kernels, is the sigmoid kernel; however, this kernel function has two parameters, which increases the modeling runtime and the kernel may not be valid for some values of the parameters.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAE</td>
<td>Absolute average error</td>
</tr>
<tr>
<td>MAE</td>
<td>Maximum absolute error</td>
</tr>
<tr>
<td>MLP</td>
<td>Multilayer perceptron</td>
</tr>
<tr>
<td>R</td>
<td>Correlation coefficient</td>
</tr>
<tr>
<td>RBF</td>
<td>Radial basis function</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root mean square error</td>
</tr>
<tr>
<td>ŷ</td>
<td>Estimated output value</td>
</tr>
<tr>
<td>αᵢ, αᵢ*</td>
<td>Lagrangian multipliers</td>
</tr>
<tr>
<td>ξᵢ, ξᵢ*</td>
<td>Slack variables</td>
</tr>
</tbody>
</table>

**References**


