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Application of Homotopy Perturbation Method to One-dimensional Transient Single-phase EM Heating Model

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Abstract

Thermal recovery involves well-known processes such as steam injection (cyclic steam stimulation, steam drive, and steam-assisted gravity drainage), in situ combustion, and a more recent technique that consists of heating the reservoir with electrical energy. When high frequency is used for heating, it is called electromagnetic (EM) heating. The applications of EM heating for heavy-oil reservoirs can be especially beneficial where conventional methods cannot be used because of large depth, reservoir heterogeneity, or excessive heat losses. This process can be modeled to determine temperature distribution in the porous reservoir rock during EM heating. In this paper, the homotopy perturbation method (HPM), a powerful series-based analytical tool, is used to approximate the temperature distribution, which has been modeled using a partial differential equation and special assumptions when high frequency currents are used. This method decomposes a complex partial differential equation to a series of simple ordinary differential equations which are easy to solve. According to the comparison of the solutions obtained by HPM with those of a numerical method (NM), good agreement is achieved. Moreover, a sensitivity study is done to determine the effect of initial temperature, oil rate, frequency and input power on the accuracy of HPM.

Keywords: Thermal Recovery, Steam Injection, Steam-assisted Gravity Drainage, Electromagnetic Heating, Homotopy Perturbation Method

1. Introduction

High viscosity is a major concern for the recovery from heavy-oil reservoirs. A very high oil viscosity results in a project being technically challenging, and sometimes uneconomic. Introducing heat to the formation has proven to be an effective way of lowering the oil viscosity by raising the temperature in the formation (Carrizales, 2010).

Thermal recovery involves well-known processes such as steam injection (cyclic steam stimulation, steam drive, and steam assisted gravity drainage), in situ combustion, and a more recent technique that consists of heating the reservoir with electrical energy. Among these processes, steam injection leads in development and application (Carrizales, 2010). However, there are many reservoirs where steam injection has proven to be uneconomic and unsuccessful. Examples are low permeability reservoirs (< 10 md) and extra-heavy oil reservoirs (< 10 ° API). These reservoirs represent a potential target for an alternative method such as electromagnetic (EM) heating that allows producing oil in a continuous and profitable manner (Carrizales et al., 2010).

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Electrical heating can be divided into low-frequency heating, also known as resistive heating, and electromagnetic (EM) heating (Bridges, 1985; Baylor, 1990; Sahni, 2000; Ovalles, 2002). EM heating refers to high-frequency heating. Radio-frequency (RF) and microwave (MW) heating are examples. This frequency spectrum was chosen mainly for its advantages of high temperatures reached near the wellbore zone with the subsequent effective viscosity reduction, faster heating rates, and the achievement of an optimum EM absorption coefficient that leads to a further productivity improvement (Chakma et al., 1992; Carrizales et al., 2008). EM heating does not require a heat transporting fluid such as steam or a hot fluid injection process, which avoids the complications associated with generating and transporting a heated fluid and allows it to be applied in wells with low incipient injectivity (Carrizales, 2010). Although formation heating at high frequency is technically feasible, there are some potential disadvantages, namely the relatively large cost of the capital equipment required to convert 60 Hz power to high frequency and the decrease in wavelength and distance to which EM energy penetrates into formation as frequency is increased (Vermeulen et al., 2000).

Several authors have dealt with the possibility of using EM heating to enhance recovery from heavy oil reservoirs (Abernethy, 1976; Vermeulen et al., 1983; McPherson et al., 1985; Kim, 1987; Fanchi, 1990). Kim (1987) developed a numerical simulator using finite differencing to study the production response of a reservoir when EM heating is applied. He included the flow of oil, water, and steam in the formulation; however, the results presented are scarce. Fanchi (1990) presented a model for determining the temperature increase associated with EM heating. Although multiple phases are accounted for calculating the total energy gained by a reservoir undergoing EM heating, his model neglects any phase changes (no heat of vaporization). Sumbar et al. (1992) developed a numerical model to simulate single-phase EM heating which was used to evaluate schemes for EM heating of heavy oil reservoirs to enhance production. Ovalles et al. (2002) coupled the Lambert's equation to a commercial simulator in order to model reservoir dielectric heating. All the above studies reported acceleration in the oil production rate compared to initial cold production. Dabletbaev et al. (2010) extended a mathematical model of the RF-EM stimulation technique based on some field applications in Russia. Carrizales et al. (2010) presented a multiphase, two-dimensional radial model which described the three-phase flow of water, oil, and steam and heat flow in a reservoir with confining formations and included gravity effects and vertical heat loss into the confining layers. A single well was used to locate the EM source and also to produce oil. They used COMSOL Multiphysics to solve the model for temperature distribution and then productivity improvement. Their model also accounted for the appearance and/or disappearance of a phase and used the variation in temperature and water saturation to continuously update the EM heating rate.

Finding temperature distribution is an important step in all the above models. In solving these partial differential equations (PDE) by numerical methods, stability and convergence should be considered. Otherwise, solutions lead to inappropriate results .A semi-exact method called homotopy perturbation has recently been established and many authors have used this technique for solving nonlinear equations (El-shahed, 2005; Cai et al., 2006; Cveticanin, 2006; Abbasbany, 2006; He, 2006; Rafei, 2006; Ganji et al., 2006; Ganji, 2006; Belendez et al., 2007). The homotopy perturbation method (HPM) is a powerful series-based analytical tool which decomposes a complex partial differential equation to a series of simple ordinary differential equations which are easy to solve. More details of this method are discussed later in this paper.

In this study, homotopy perturbation method (HPM) is deployed to solve a nonlinear PDE which describes a special case of Carrizales's model which is a one-dimensional, transient, single-phase EM heating model (Carrizales et al., 2010). In addition, this equation is solved by a numerical method and the results are compared with those obtained by HPM. Moreover, a sensitivity study is done to determine the effect of initial temperature, oil rate, frequency, and input power on the accuracy of HPM.

2. Basic concept of homotopy perturbation method

To explain the basic idea of the HPM for solving nonlinear differential equations, the following nonlinear differential equation is considered:

$$A(u) - f(r) = 0, \quad r \in \Omega \tag{1}$$

The equation is subject to the following boundary condition:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma \tag{2}$$

where, A is a general differential operator; B is a boundary operator; $f(r)$ stands for a known analytical function; Γ represents the boundary of domain (Ω) and $\partial u / \partial n$ denotes differentiation along the normal drawn outwards from Ω. The operator A can, generally speaking, be divided into two parts: a linear part L and a nonlinear part N. Equation 1, therefore, can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0 \tag{3}$$

In case the nonlinear Equation1 has no "small parameter", we can construct the following homotopy:

$$H(v, P) = (1 - P)[L(v) - L(u_0)] + P[L(v) + N(v) - f(r)] = 0 \tag{4}$$

where, P is called homotopy parameter and $u_0$is an initial approximation which satisfies the boundary conditions. According to the HPM, the approximate solution of Equation 4 can be expressed as a series of the power of P, i.e.:

$$v = v_0 + Pv_1 + P^2v_2 + \cdots, \tag{5a}$$

$$u = \lim_{P\to 1} v = v_0 + v_1 + v_2 + \cdots. \tag{5b}$$

When Equation 4 corresponds to Equations 1, Equation 5b becomes the approximate solution of Equation 1. Some interesting results have been obtained using HPM (Ganji et al., 2009; Babolian et al., 2009; Hemeda, 2012; Cveticanin, 2006; El-shahed, 2005; Ganji et al., 2006; He, 2005; He, 2005). Ganji (2006) compared homotopy perturbation method (HPM) with numerical methods in the heat transfer field. To do this, he considered two nonlinear PDE's related to two cases, namely cooling of a lumped system with variable specific heat and the temperature distribution equation in a thick rectangular fin radiation to free space. He showed that He's homotopy perturbation method (HPM) completely overcame the inaccurate results obtained by numerical methods especially in cases where the equation was intensively dependent on time. Ganji et al. (2006) attempted to show the capabilities and wide-range applications of the homotopy perturbation method in comparison with the previous ones in solving heat transfer problems. In their research, homotopy perturbation method was used to solve an unsteady nonlinear convective-radiative equation and a nonlinear convective-radiative conduction equation. They also showed that HPM had a small error compared to the exact solution as the rate of nonlinearity was higher. Cveticanin (2006) used the homotopy perturbation method proposed by J.-He. He to solve pure strong nonlinear second-order differential equations. Two types of differential equations were considered, i.e. with strong cubic and strong quadratic nonlinearity. The obtained solution was compared with the exact numerical one. The difference between these solutions was negligible for a long-time period. The method was found to work extremely well in the examples.

Siddiqui et al. (2008) analyzed the thin film flow problem with a third grade fluid on an inclined plane. The governing non-linear equation was solved for the velocity field using the traditional perturbation technique as well as homotopy perturbation method and the results were compared; they were in complete agreement. Biazar et al. (2009) presented the use of the He's homotopy perturbation method for the systems of linear and non-linear Volterra integral equations of the second kind. For linear and non-linear systems, very good approximations of the solutions were obtained. They concluded that the He's homotopy perturbation method was a powerful and efficient technique in finding very good solutions to this kind of systems. Belendez et al. (2007) used homotopy perturbation method to solve the nonlinear differential equation governing the nonlinear oscillations of a system typified as a mass attached to a stretched elastic wire. It was found that this perturbation method worked very well for the whole range of parameters involved, and excellent agreement of the approximate frequencies and periodic solutions with the exact ones could be obtained. Cveticanin (2009) used He's homotopy perturbation method to solve non-linear partial differential equations. An approximate solution to the differential equation describing the longitudinal vibration of a beam was obtained. The solution was compared with the one found using the variational iteration method introduced by He. The difference between the two solutions was negligible. Ravi Kanth et al. (2009) tried to find the numerical solution of linear and non-linear higher-order boundary value problems using He's homotopy perturbation method. This technique was tested on three examples and was seen to produce satisfactory results. Cai et al. (2009) applied the homotopy perturbation method to nonlinear oscillations. It was demonstrated that the obtained insightful solutions were of high accuracy even with the first-order approximation. Fathizadeh et al. (2009) solved convective heat transfer equations of boundary layer with pressure gradient over a flat plate using homotopy perturbation method (HPM). They showed that results agreed well with those obtained numerically. Yildirim et al. (2012) solved the steady two-dimensional laminar forced magnetohydrodynamic Hiemenz flow against a flat plate with a variable wall temperature in a porous medium using the homotopy perturbation method (HPM). The skin friction coefficient and the rate of heat transfer given by the HPM were in good agreement with the numerical solutions of the Keller box method.

It should be pointed out that the following benefits have been suggested by different authors (Ganji, 2006; Cveticanin, 2006; Belendez et al., 2007; Cai et al., 2009; Fathizadeh et al., 2009):

1. The HPM is valid for all the nonlinear equations with a high order of nonlinearity containing different parameters;
2. The suggested method works well due to the fact that it uses the advantages of the homotopy, perturbation, and power series expansion methods;
3. This technique yields a very rapid convergence of the solution series; in most cases, only one iteration leads to the high accuracy of the solution;
4. Due to the simplicity and accuracy of the He's homotopy perturbation method, the application of this method is recommended when solving practical technical problems. Both the reliability of the method and the possibility of using computers to obtain a more accurate solution allow the method to be widely applied;
5. The reliability of the method and the reduction in the size of computational domain render this method a wider applicability.

3. Governing equations

Equation 6 represents a general form of the total energy balance for a reservoir under EM heating, including energy transport by convection, conduction, and the EM heating source for any number of phases. Once the EM heating source term is determined and substituted into Equation 6, according to the corresponding coordinate system used, the temperature distribution of a reservoir undergoing EM heating can be estimated (Carrizales, 2010).

$$\frac{\partial T}{\partial t}\left(\emptyset \sum_{j=1}^{N_p} M_j S_j + (1-\emptyset)M_s\right) + \emptyset \sum_{j=1}^{N_p} H_j \frac{\partial}{\partial t}(\rho_j S_j) + \left(\sum_{j=1}^{N_p} M_j \bar{u}_j\right).\bar{\nabla}T + \sum_{j=1}^{N_p} H_j \nabla.(\rho_j \bar{u}_j) - \bar{\nabla}.\left(k_{T_{eff}}\bar{\nabla}T\right) = -\nabla.\bar{q}_{EM} \quad (6)$$

The energy contribution because of the EM source applied to a radial system can be expressed as (Carrizales, 2010).

$$\nabla.\bar{q}_{EM} = -\alpha\frac{P_0 e^{-\alpha(r-r_w)}}{r} \quad (7)$$

where, T, t, $\emptyset$, N_p, and M_j are temperature, time, porosity, the number of phases, volumetric heat capacity of phase j respectively. S_j, M_s,H_j, ρ_j, and $\bar{u_j}$ stand for the saturation of phase j, the volumetric heat capacity of rock, the enthalpy of phase j, the density of phase j, and Darcy velocity vector of phase j respectively. k_{Teff} is effective thermal conductivity and $\overline{q_{EM}}$ is energy input from EM heating source; α represents EM absorption coefficient, and P_0 is the incident power radiated at the wellbore; r and r_w are radial distance and the wellbore radius respectively.

The total conservation of energy for radial single-phase, one-dimensional counter-current (a single well is used for production and setting antenna for EM heating) transient flow is obtained from Equation 6 as reads (Carrizales, 2010).

$$M_T\frac{\partial T}{\partial t} = -M_o u_o\frac{\partial T}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(k_{Teff} r\frac{\partial T}{\partial r}\right) + \alpha\frac{P_0 e^{-\alpha(r-r_w)}}{2\pi hr} \quad (8)$$

$$M_T = \emptyset M_w + \emptyset M_o + (1-\emptyset)M_s \quad (9)$$

where, M_T is total volumetric heat capacity and h is pay zone thickness. All the other parameters are as defined above.

To solve Equation 8, the following initial and boundary conditions are used (Carrizales, 2010):

$$T = T_i, \quad t = 0, \quad r_w < r < r_e \quad (10)$$

$$\frac{\partial T}{\partial r} = 0, \quad t > 0, \quad r = r_w \quad (11)$$

$$T = T_i, \quad t > 0, \quad r = r_e \quad (12)$$

where, r_e is the drainage radius of the reservoir and T_i represents the initial temperature of the reservoir.

4. Results and discussions

4.1. HPM solution for temperature distribution

HPM is applied herein to solve Equation 8. To this end, after some mathematical manipulations, we construct a homotopy of the nonlinear partial differential equation (Equation 8) as reads:

$$(1-P)\left(-M_T\frac{\partial T}{\partial t} + M_T\frac{\partial T_0}{\partial t}\right) + P\left(-M_T\frac{\partial T}{\partial t} + \left(-M_o u_o + \frac{k_{Teff}}{r}\right)\frac{\partial T}{\partial t} + k_{Teff}\frac{\partial^2 T}{\partial r^2} + \alpha\frac{P_0 e^{-\alpha(r-r_w)}}{2\pi hr}\right) = 0 \quad (13)$$

We consider the approximate solution as follows:

$$T = T_o + PT_1 + P^2 T_2 + \cdots \quad (14)$$

For the ease of mathematical calculations, terms after P^2 term are truncated in Equation 14. Assuming $\frac{\partial T_0}{\partial t} = 0$, substituting T from Equation 14 into Equation 13, applying some simplifications, and rearranging based on the powers of P-terms, one may obtain:

$$P^0: -M_T\frac{\partial T_0}{\partial t} = 0 \tag{15}$$

$$T_0(t, r) = T_i \tag{16}$$

$$P^1: -M_T\frac{\partial T_1}{\partial t} + Ar^{-1}\frac{\partial T_0}{\partial r} + k_{Teff}\frac{\partial^2 T_0}{\partial r^2} + Br^{-1}e^{-\alpha(r-r_w)} = 0 \tag{17}$$

$$T_1(t, r) = 0 \tag{18}$$

$$P^2: -M_T\frac{\partial T_2}{\partial t} + Ar^{-1}\frac{\partial T_1}{\partial r} + k_{Teff}\frac{\partial^2 T_1}{\partial r^2} = 0 \tag{19}$$

$$T_2(t, r) = 0 \tag{20}$$

$$A = -\frac{M_o q_o}{2\pi} + k_{Teff} \tag{21}$$

$$B = \frac{\alpha P_0}{2\pi h} \tag{22}$$

where, M_o is oil volumetric heat capacity and q_o stands for oil rate which is constant in this study. Solving Equations 15-19 and applying boundary conditions (Equations16-20), one may have:

$$T_0 = T_i \tag{23}$$

$$T_1 = \frac{B}{M_T} r^{-1} e^{-\alpha(r-r_w)} t \tag{24}$$

$$T_2 = \frac{Bt^2}{2{M_T}^2} e^{-\alpha(r-r_w)}(r^{-3}(2k_{Teff} - A) + r^{-2}(2k_{Teff}\alpha - A\alpha) + r^{-1}k_{Teff}\alpha^2) \tag{25}$$

According to Equation 14 and assuming P =1, we get the following approximation for the temperature distribution:

$$T = T_i + \frac{B}{M_T} r^{-1} e^{-\alpha(r-r_w)} t + \frac{Bt^2}{2{M_T}^2} e^{-\alpha(r-r_w)}(r^{-3}(2k_{Teff} - A) + r^{-2}(2k_{Teff}\alpha - A\alpha) + r^{-1}k_{Teff}\alpha^2) \tag{26}$$

Since Equation 8 cannot be easily solved by analytical methods, this equation is, therefore, solved by a numerical method applying MATLAB using function *pdepe* which is based on finite difference method. For the numerical calculations, data in Table 1 are used. The values of temperature obtained by numerical method and HPM, for a specific time and at different locations (r) in the reservoir, are given in Figure 1. As can be seen, HPM has a high accuracy at early times.

Table 1
Input parameters for the determination of temperature distribution

M_o	2.046×106 J.m-3.k-1	P_0	63 kW
M_T	2.314×106 J.m-3.k-1	h	91.44m
q_o	0.199 m3/hr	T_i	366.483 K
k_{Teff}	2.999 $W.m^{-1}.k^{-1}$	Ø	0.110
α	0.133 m^{-1}	r_w	0.178m

Also, the other results of the two different methods of homotopy perturbation and numerical method for temperature distribution are compared in Figures 1c and 1d, which indicates less accurate

agreement between two methods in some distances from wellbore compared to the previous figures. As mentioned before, HPM gives an approximation to the solution to differential equations. Usually, taking more terms in the series of the power of P into account gives higher accuracy in HPM.
The reason for the less accurate solutions in the cases of t =15 days and t =20 days compared to the previous cases (Figures 1a and 1b) is that the series are truncated after the P^2-term. This problem can be solved by considering more terms in the series of the power of P. For example, Figure 2 compares the temperature distribution at t =5 hours when one additional power P term is considered in Equation 26. This term is written below:

$$T_3 = \frac{Bt^3}{6M_T{}^3} e^{-\alpha(r-r_w)}(r^{-5}(AF - 4k_{Teff}F) + r^{-4}(AC + k_{Teff}J) + r^{-3}(AD + k_{Teff}G) + r^{-2}(-AE + \text{kTeffH}+\text{r}-\text{1kTeffJ}) \quad (27)$$

where,

$$C = 3\alpha(A - 2k_{Teff}) \quad (28)$$

$$D = \alpha^2(A - 3k_{Teff}) \quad (29)$$

$$E = k_{Teff}\alpha^3 \quad (30)$$

$$F = -6k_{Teff} + 3A \quad (31)$$

$$G = -\alpha C - 2D \quad (32)$$

$$H = -\alpha D + E \quad (33)$$

$$I = \alpha E \quad (34)$$

$$J = -\alpha F - 3C \quad (35)$$

As can be seen, the accuracy of HPM becomes better compared to the previous case. It should be noted that care should be taken that this additional term at times longer than 5 hours may results in unreliable answers; this means that for these longer times, greater powers of P should be considered. Furthermore, this disagreement between HPM and numerical methods arises, in this study, for small distances from wellbore, which can be overcome by the same method that is considering more terms in the series of power of P.

4.2. Effects of frequency, oil rate, initial temperature, and input power on HPM accuracy

Figures 3-6 illustrate the effects of frequency, oil rate, input power, and initial temperature on HPM accuracy when t =5 days and the data in Table 1 are used.

a. Effect of frequency

The EM frequency was varied in a range of 13.46 to 2450 MHz to examine its effect on the HPM accuracy. Figure 3 shows the temperature distribution when different EM frequencies were used for EM heating. The higher the frequency is, the less the HPM accuracy becomes.

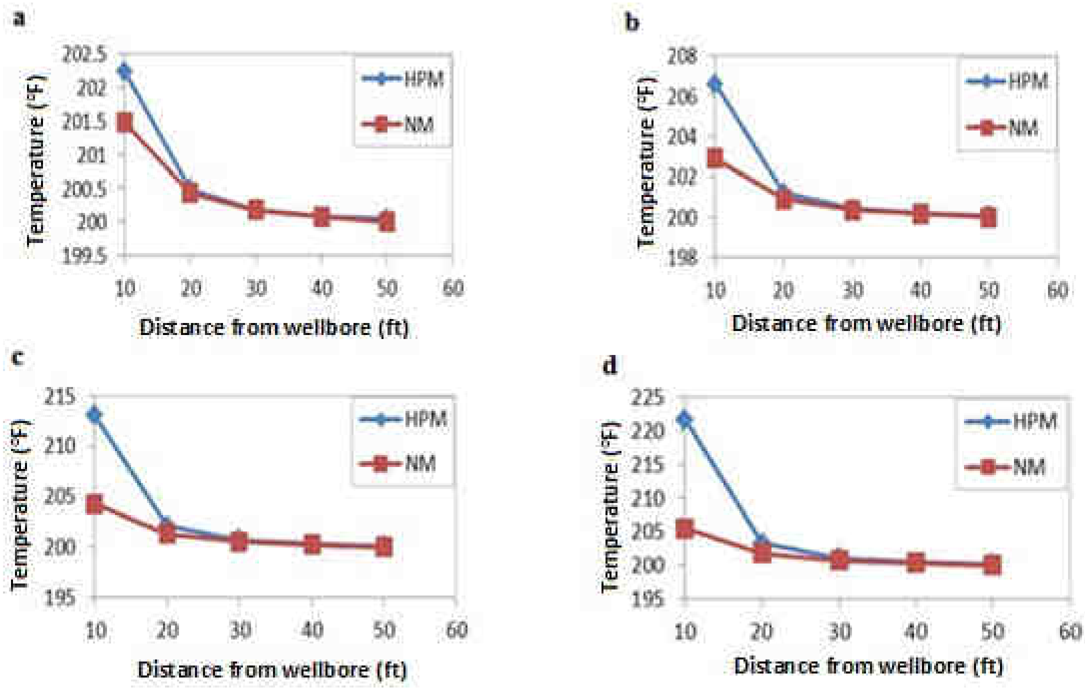

Figure 1
Temperature distribution as a function of distance from wellbore at (a) t =5 days, (b) t =10 days, (c) t =15 days, and (d) t =20 days

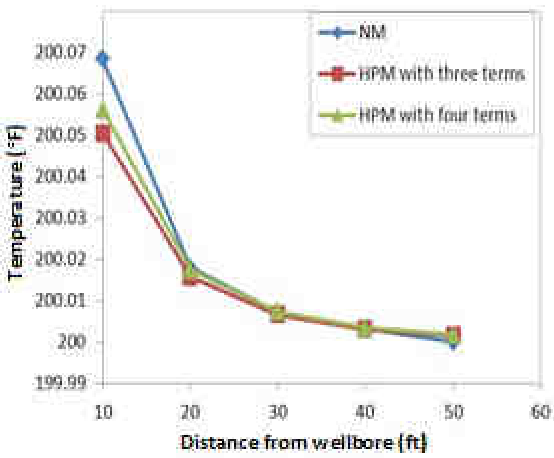

Figure 2
Temperature distribution as a function of distance from wellbore at t =5 hours when an additional term is added to temperature distribution equation.

b. Effect of oil rate

To evaluate the effect of oil rate on the HPM accuracy, several runs were conducted using different values for this parameter. Figure 4 shows a comparison of HPM accuracy for oil rates from 10 to 100 barrels per day. As oil rate increases, the HPM accuracy decreases.

c. Effect of input power

The effect of input power on the HPM accuracy was studied in three different cases at powers of 5, 50, and 150 kW. As can be seen from Figure 5, the results show that the HPM accuracy increases as the input power source is decreased.

d. Effect of initial temperature

To evaluate the effect of the initial temperature of the reservoir on the HPM accuracy when EM heating was applied various temperature distributions using initial temperatures from 100 to 300 °F were obtained and compared. Figures 6a-c show the comparison of the HPM accuracy for different initial temperatures. The results show that the initial temperature has no remarkable effect on the HPM accuracy. Equation 26 can also explain why initial temperature does not change the HPM accuracy.

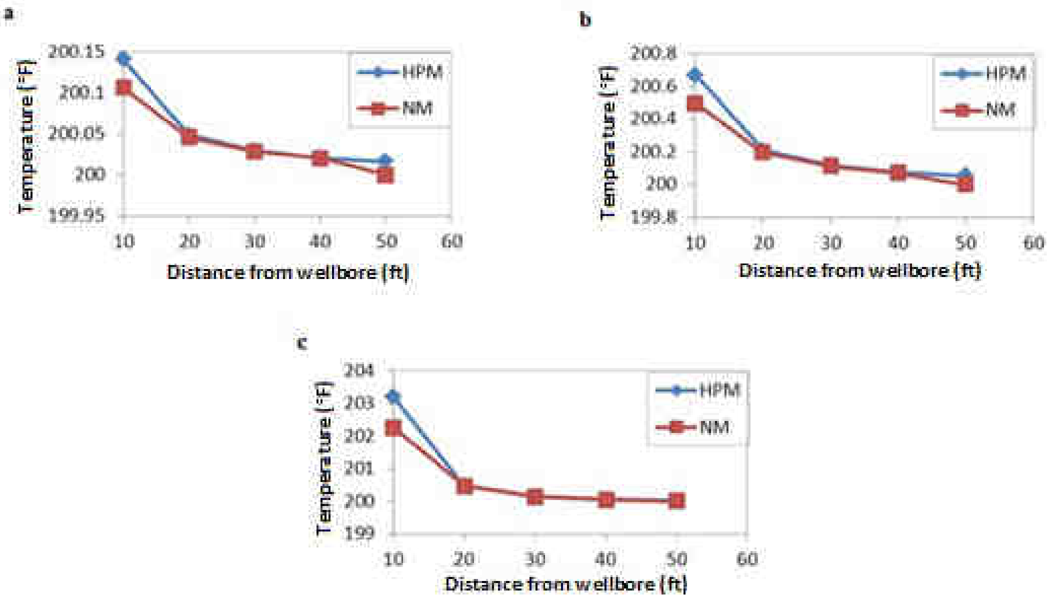

Figure 3
Temperature distribution as a function of distance from wellbore at t =5 days at (a) frequency=13.54 MHz, (b) frequency=140.6 MHz, and (c) frequency=2450 MHz

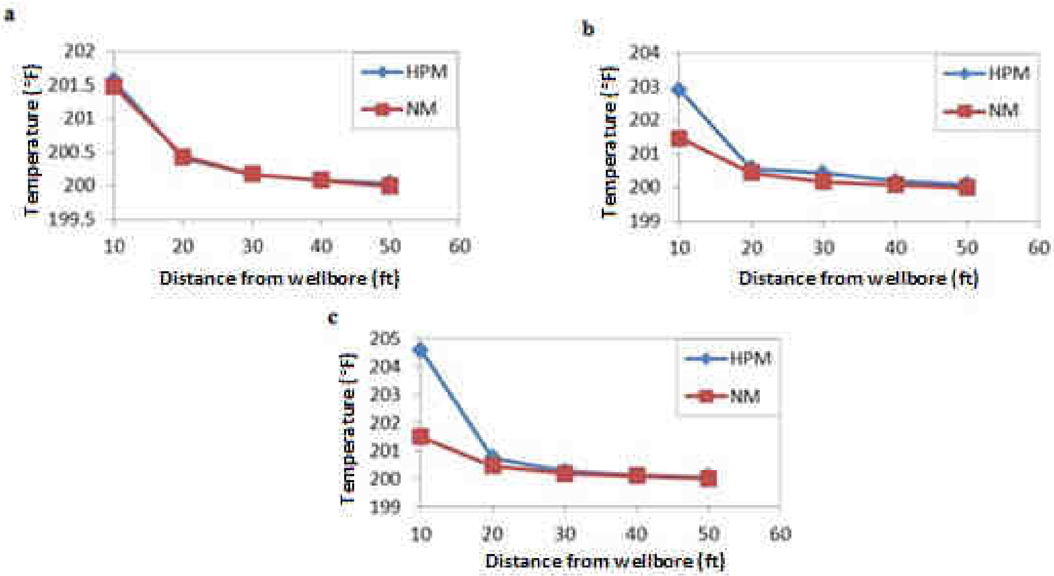

Figure 4
Temperature distribution as a function of distance from wellbore at t =5 days at (a) q_o =10 barrels per day, (b) q_o =50 barrels per day, and (c) q_o =100 barrels per day

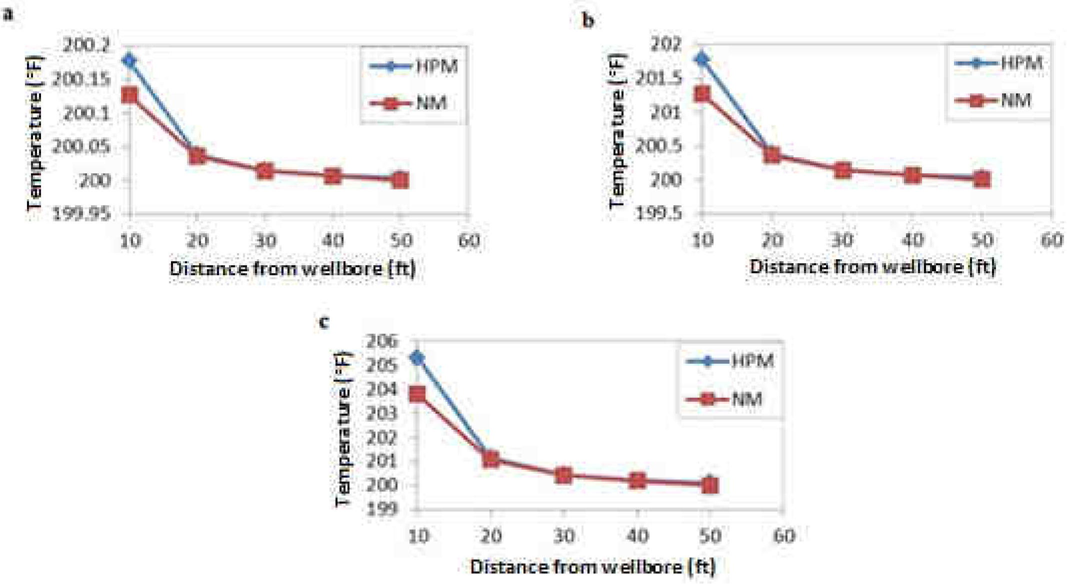

Figure 5
Temperature distribution as a function of distance from wellbore at t =5 days at (a) P_0 =5 kw, (b) P_0 =50 kw, and (c) P_0 =150 kw

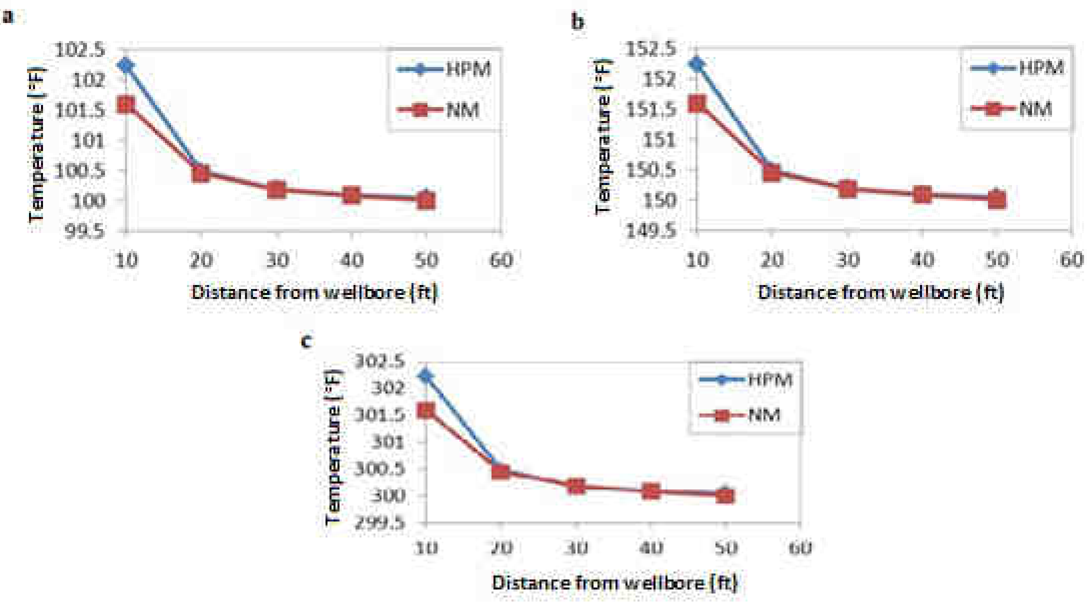

Figure 6
Temperature distribution as a function of distance from wellbore at t =5 days at (a) T_i =100 °F, (b) T_i =150 °F, and (c) T_i =300 °F

5. Conclusions

In this study, HPM is used for solving the temperature distribution from one-dimensional, transient, single-phase EM heating model. The homotopy perturbation method is appropriate for solving strong non-linear partial differential equations. The strong non-linear partial differential equations are transformed into a system of ordinary differential equations which are suitable for calculation. To show the accuracy of the HPM, a governing equation is simultaneously solved by a numerical method using MATLAB. The results show that HPM has high accuracy at early times of EM heating process.

This accuracy can also be high in middle- and late-time regions by considering further terms in the series of the power of P in HPM. Moreover, it is shown that parameters such as oil rate, input power, and frequency have an inverse relation with HPM accuracy and the initial temperature of the reservoir has no remarkable effect on HPM accuracy. He's homotopy perturbation method can be a reliable tool for solving partial differential equations describing different engineering problems. Both the reliability of the method and the possibility of using computers to obtain an accurate solution allow the method to be widely applied.

Nomenclature

A	: General differential operator
B	: Boundary operator
$f(r)$	: Known analytical function
h	: Pay zone thickness
H_j	: Enthalpy of phase j
k_{Teff}	: Thermal conductivity
L	: Linear part of operator A
M_j	: Volumetric heat capacity of phase j
M_o	: Oil volumetric heat capacity
M_s	: Volumetric heat capacity of rock
M_T	: Total volumetric heat capacity
N	: Nonlinear part of operator A
N_p	: Number of phases
P	: Homotopy parameter
P_0	: Incident power radiated at the wellbore
$\overline{q_{EM}}$	: Energy input from electromagnetic heating source
q_o	: Oil rate
r	: Radial distance
r_e	: Drainage radius of the reservoir
r_w	: Wellbore radius
s_j	: Saturation of phase j
t	: Time
T	: Temperature
T_i	: Initial temperature of the reservoir
T_j(j=1, 2, 3,…)	: Series terms of Equation 14
u	: Solution of Equation 1
u_j(j=1, 2, 3,…)	: Series terms of Equation 5a
$\overline{u_j}$	: Darcy velocity vector of phase j
v	: Solution of Equation 4
v_j(j=1, 2, 3,…)	: Series terms of Equation 5b
Greek symbols	
α	: Electromagnetic absorption coefficient
Γ	: Boundary of domain Ω
ρ_j	: Density of phase j
$\emptyset$	: Porosity
Ω	: Domain
$\partial u/\partial n$	: Differentiation along normal drawn outwards from Ω
Subscripts	
0	: Incident
e	: Drainage

EM	: Electromagnetic
i	: Initial
j	: Phase
o	: Oil
p	: Phase
s	: Rock
T	: Total
T_{eff}	: Effective temperature
W	: Wellbore

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