

Characterization of Liquid Bridge in Gas/Oil Gravity Drainage in Fractured Reservoirs

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Abstract

Gravity drainage is the main mechanism which controls the oil recovery from fractured reservoirs in both gas-cap drive and gas injection processes. The liquid bridge formed between two adjacent matrix blocks is responsible for capillary continuity phenomenon. The accurate determination of gas-liquid interface profile of liquid bridge is crucial to predict fracture capillary pressure precisely. The liquid bridge interface profile in the absence and in the presence of gravity is numerically derived, and the obtained results are compared with the measured experimental data. It is shown that in the presence of gravity, fracture capillary pressure varies across the fracture, whereas, by ignoring gravitational effects, a constant capillary pressure is obtained for the whole fracture. Critical fracture aperture which is the maximum aperture that could retain a liquid bridge was computed for a range of liquid bridge volumes and contact angles. Then, non-linear regression was conducted on the obtained dataset to find an empirical relation for the prediction of critical fracture aperture as a function of liquid bridge volume and contact angle. The computation of fracture capillary pressure at different liquid bridge volumes, fracture apertures, and contact angles demonstrates that if the liquid bridge volume is sufficiently small (say less than 0.5 microliters), capillary pressure in a horizontal fracture may reach values more than 0.1 psi, which is comparable to capillary pressure in the matrix blocks. The obtained results reveal that the variation of fracture capillary pressure versus bridge volume (which represents liquid saturation in fracture) obeys a trend similar to the case of matrix capillary pressure. Therefore, the capillary pressure of matrix can be applied directly to fractures considering proper modifications. The results of this study emphasize the importance of capillary continuity created by liquid bridges in the performance of gas-oil gravity drainage in fractured reservoirs.

Keywords: Fractured Reservoir, Gravity Drainage, Capillary Continuity, Liquid Bridge, Fracture Capillary Pressure

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1. Introduction

Gas injection in fractured reservoirs is believed to be able to enhance oil recovery significantly (Montazeri et al., 2017). Gravity drainage is known to be the main production mechanism of gas-cap drive fractured reservoirs as well as fractured reservoirs subjected to gas injection. Under such conditions, oil saturated matrix blocks will be surrounded by gas-filled fractures. Those matrix blocks surrounded by gas will undergo a gravity drainage process if gravitational forces exceed capillary forces (Sajjadian et al., 1999). Several experimental studies and field observations revealed that the efficiency of gravity drainage process is largely affected by the interaction of neighboring matrix blocks, namely block-to-block interaction. Block-to-block interaction can be described by the combination of two different phenomena: capillary continuity between blocks and reinfiltration of the drained liquid from upper to lower blocks (Dejam et al., 2014). The existence of capillary continuity between neighboring matrix blocks increases the height of the continuous vertical liquid column of fractured reservoirs, thereby improving final recovery drastically. Both laboratory experiments and field observations of some fractured reservoirs indicate that defining matrix blocks as discontinuous blocks is not realistic (Horie et al., 1990). Three different mechanisms are believed to be responsible to create capillary continuity between matrix blocks: continuity through porous spacers in fracture, continuity via thin liquid films around non-porous spacers, and continuity through liquid bridges in fracture. In real fractured porous media, neighboring matrix blocks may have several contacts due to natural asperities on fracture surface. The existence of porous spacers in fractures mimics this type of continuity. On the other hand, when fracture aperture is smaller than a critical value, the travelling liquid thread draining from upper matrix block reaches the upper face of lower matrix block before detachment. Consequently, a liquid bridge forms in the fracture between two matrix blocks. Festoy and van Golf-Racht (1989) argued that a model of a fractured reservoir, in which matrix blocks are separated by fractures, but some additional connections exist between them, is more representative of fractured reservoirs. They used a conventional simulator to evaluate the influence of contact area on gravity drainage rate and ultimate recovery. Their simulation results showed that a limited contact area (about 25% and more) between matrix blocks is able to create complete capillary continuity. Similar observations were reported in the experiments performed by Labastie (1990). He performed gravity drainage experiments in a stack of two matrix blocks using porous and non-porous spacers in the fracture. Labastie (1990) believed that capillary continuity usually exists, but the flow rates may be small. The area, number, and permeability of porous contacts determine the rate of drainage from stacked blocks connected by porous materials. Horie et al. (1990) performed gravity drainage experiments in a stack of matrix blocks. They examined the effect of using different types of spacers in the fractures on the performance of capillary continuity. They reported that strong capillary continuity is observed between adjacent matrix blocks, so they proposed the incorporation of capillary continuity in the numerical simulation of fractured reservoirs. Saidi and Sakhtikumar (1993) believed that capillary continuity is not the case in several fractured reservoirs. As an example of a well-fractured reservoir, which indicates negligible capillary continuity, they referred to the production performance of Haftkel field (in the south-west of Iran). However, Saidi (1987) argued that if the fracture system consists of thin fractures (with an aperture of about 10 micrometers or less), capillary continuity can be realized. On the other hand, for fractures with an aperture greater than 50 micrometers as well as fractures which are filled with impermeable materials, matrix blocks may be assumed discontinuous. Firoozabadi and Markeset (1994) emphasized on the concept of gas/oil flow in the fracture medium as a critical parameter to the performance of fractured reservoirs. They performed gravity drainage experiments in three-block and four-block stack assemblage covering fracture apertures ranging from 50 to 1200 micrometers. They examined the effect of number and thickness of fracture spacers on the liquid transmissibility across the fracture for different assemblages of stacking matrix blocks. They observed that liquid transmissibility is not a function of

the number of contact points or contact area, but it is very sensitive to fracture aperture. Sajjadian et al. (1999) performed gravity drainage experiments using a stack of two sandstone blocks to investigate the strength of capillary interaction. Porous and non-porous spacers at two different thicknesses were implemented to clarify the influence of the type and thickness of fracture spacers on capillary continuity. They concluded that if fracture aperture is greater than the critical fracture aperture, which is a function of fracture and fluid properties, continuity is maintained through a film flow which is almost non-effective; as a consequence, matrix blocks are discontinuous in terms of capillary. On the other hand, when fracture aperture is sufficiently small, liquid bridges are responsible to keep capillary continuity. In the case of the existence of liquid bridges, as well as porous spacers in the fracture, an efficient continuity occurs due to high transmissibility (or bulk flow) through liquid bridges or porous spacers. The low transmissibility of the film flow when the fracture aperture is large was also mentioned by Firoozabadi and Markeset (1994).

The characteristics of liquid bridges between two rigid bodies is of key importance in many industrial applications, including soil mechanics, granular materials, pharmaceuticals, geophysics, geoenvironment, and oil recovery from fractured reservoirs. The formation, stability, and properties of liquid bridges have been investigated in several theoretical and experimental studies. However, the published literature is limited with respect to the aim of this paper, i.e. the formation of liquid bridge in fractures and its role in maintaining capillary continuity. Dejam and Hassanzadeh (2011) developed a theoretical model for the description of the elongation and subsequent breakup of a liquid element suspended from a horizontal surface; the horizontal surface mimics the bottom face of a matrix block. The developed model was coupled with an approximation of Young-Laplace Equation (YLE) to find an expression that relates fracture capillary pressure to the critical length of liquid element at the onset of liquid bridge formation. Their approximation of YLE ignored gravity effects and assumed the gas-liquid interface as a part of a circle (i.e. Toroid approximation). Mashayekhizadeh et al. (2012) considered the stability of liquid bridges in fractured porous media on the pore scale using a glass micromodel representing a stack of two blocks at different tilt angles to monitor the stability of liquid bridges and the shape of a front during a free gravity drainage process. Although the process of liquid bridge formation and breakup was observed during gravity drainage experiments performed by Sajjadian et al. (1999), no attempt was made to analyze liquid bridge characteristics theoretically. Dejam et al. (2014) studied the process of liquid reinfiltration through liquid bridges in fractures between two porous matrix blocks. They used a generalized form of the Lucas–Washburn theory for a porous medium to find out a numerical solution for the prediction of the depth and rate of reinfiltration. In another work, Dejam et al. (2014) presented a numerical solution of YLE which determined the gas-liquid interface of a liquid bridge between two parallel plates. They neither examined the effect of gravity nor the stability criteria of liquid bridge. In addition, no effort was made to relate capillary pressure to liquid bridge volume or liquid saturation.

The present study is focused on the development of a reliable model to predict liquid bridge characteristics, particularly liquid bridge shape, its capillary pressure, and its stability. The accurate prediction of gas-liquid interface of liquid bridge is a key factor in the precise prediction of fracture capillary pressure. The clarification of the interdependency of liquid bridge properties and fracture capillary pressure is helpful in the interpretation of capillary pressure behavior in a fracture network. To this end, Young-Laplace equation is adopted for different cases of liquid bridge in horizontal fractures, and a numerical solution is presented. Then, by comparing experimental results with the developed numerical model, its accuracy is checked. Thereafter, the developed model is implemented to predict liquid bridge properties. Furthermore, the determination of necessary condition for the stability of liquid bridges is the other goal of this paper. The critical fracture aperture, i.e. the maximum

fracture aperture through which a liquid bridge can exist, could be determined for liquid bridge at different volumes and contact angles.

2. Mathematical study

This section describes different approaches to implementing numerical schemes to find out different characteristics of liquid bridge, including shape, mean curvature, and capillary pressure. To this end, liquid bridges between two flat plates and between two equal spheres are considered. First, in the case of liquid bridge between flat plates, the solution is found in the absence of gravity. Thereafter, this solution is modified to consider the gravitational effects on liquid bridge behavior. Afterwards, liquid bridge between two spheres is considered. The extension of the results of liquid bridge between spheres to liquid bridge in horizontal fractures is also discussed.

2.1. Liquid bridge between two flat plates

Figure 1 illustrates axisymmetric liquid bridges between two parallel plates and between two spheres. In this figure, b is separation distance (known as fracture aperture in this study), and θ stands for contact angle; α is half filling angle, and R represents sphere radius. If the lower face of the upper matrix block and the upper face of the lower matrix block are flat and smooth, the schematic depicted in Figure 1 (left) simulates a liquid bridge in a horizontal fracture. The Young-Laplace equation (YLE) relates the surface tension of the liquid to the pressure difference across the interface and the mean curvature of the bridge surface. The meniscus profile of a liquid bridge is evolved by minimizing its surface energy (Van Honschoten et al., 2010). The solution of YLE yields the shape of the bridge which corresponds to the minimum surface for the bridge of interest. In the absence of gravity, YLE can be expressed as follows (Adamson, 1982):

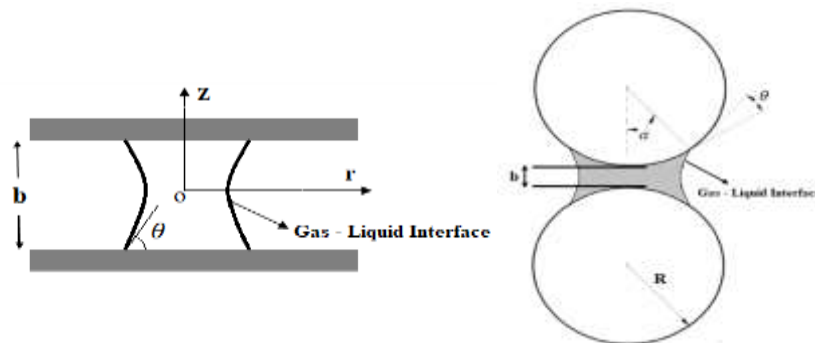


Figure 1

A schematic view of a liquid bridge between two smooth plates (left) and between two equal spheres (right); b is separation distance known as fracture aperture and θ is contact angle.

$$P_c = \sigma \left[\frac{1}{R_1} + \frac{1}{R_2} \right] \quad (1)$$

where, P_c is pressure difference across gas-liquid interface, i.e. $P_c = P_g - P_o$; σ stands for surface tension, and R_1 and R_2 are the principle radii of curvatures. For an axi-symmetric liquid bridge, the following expressions could be derived for R_1 and R_2 (see Van Honschoten et al. (2010) or Gangux and Millet (2014) for the details of derivation):

$$\frac{1}{R_1} = \frac{\frac{d^2 r}{dz^2}}{\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{\frac{3}{2}}} \quad (2)$$

and

$$\frac{1}{R_2} = \frac{1}{r \left[1 + \left(\frac{dr}{dz}\right)^2\right]^{\frac{1}{2}}} \quad (3)$$

Liquid bridge mean curvature is defined as the arithmetic average of the inverse of principle radii of curvatures. One can simply find the following relation between pressure difference (P_c) and mean curvature (H):

$$P_c = 2\sigma H \quad (4)$$

Boundary conditions for a liquid bridge between two plates may be described as:

$$r \Big|_{z=\frac{b}{2}} = r_c \quad (5)$$

$$\frac{dr}{dz} \Big|_{z=\frac{b}{2}} = \cot(\theta) \quad (6)$$

Boundary conditions given by Equations 5 and 6 are established from the contact point of liquid bridge and solid body. Boundary conditions can also be defined in the neck of bridge equivalently:

$$r \Big|_{z=0} = r_o \quad (7)$$

$$\frac{dr}{dz} \Big|_{z=0} = 0 \quad (8)$$

Substituting Equations 2 and 3 in Equation 1 yields a second order non-linear ordinary differential equation (ODE). Numerical schemes are applicable to finding the solution of this equation; we used a modified Euler method to find the numerical solution of Equation 1 with its boundary conditions expressed by Equations 5 and 6 (or 7 and 8). The details of computational procedure of the modified Euler method for a similar problem (liquid bridge between two spheres) was presented by Lian et al. (1993). The solution of the abovementioned equation, $r(z)$, represents the meniscus profile of liquid bridge as well as its mean curvature. If the infinitesimally thin vertical sections of liquid bridge are assumed to be circular, a good approximation of bridge volume can be obtained directly from the interface profile as follows:

$$V = \int_{-\frac{b}{2}}^{\frac{b}{2}} \pi r^2 dz \quad (9)$$

When no external force field (e.g. gravity) exists, pressure difference and mean curvature are uniform at any point in the bridge (Hotta et al., 1974). The influence of gravity may be neglected when the ratio of gravity force to surface tension force is sufficiently small. As a quantitative basis, Sanz and Martinez (1982) suggested that Bond number, as defined by the following equation, be a measure of the effects of gravity to surface tension of a liquid bridge.

$$Bo = \frac{\Delta\rho gLD}{\sigma} \quad (10)$$

where, $\Delta\rho$ is the density difference between liquid and surrounding gas, and L stands for a characteristic vertical length; D represents a characteristic radius of the curvature of the interface. Sanz and Martinez (1982) argued that for $Bo \ll 1$, gravity influence may be ignored in the analysis of liquid bridge. On the other hand, Schubert (1984), during the studies of the behavior of solid particulate systems, argued that, for liquid bridge between particles with sizes smaller than 1 mm, gravity effects may be ignored. However, for the case of gravity drainage in fractured reservoirs, where liquid bridge forms between matrix blocks, the above conditions ($Bo \ll 1$ and $D < 1$ mm) are not necessarily satisfied. Therefore, the role of gravity on liquid bridge characteristics, particularly on capillary pressure, needs to be addressed. When gravity must be counted, Equation 1 is modified to the following form (Hotta et al., 1974):

$$\rho g(z + z_o) = \sigma \left[\frac{1}{R_1} + \frac{1}{R_2} \right] \quad (11)$$

where, $P_o = \rho g z_o$ is the pressure difference at a reference point, usually at contact point with the lower plate. The term $\rho g(z + z_o)$ represents pressure difference across the interface along the fracture. Based on this equation, one may conclude that when gravity is considered, pressure difference and interface mean curvature vary along the bridge. Also, it is obvious that as going upward in the fracture, fracture capillary pressure increases, so the maximum fracture capillary pressure occurs at the upper end of bridge, which is the bottom of the upper matrix block. Since the liquid bridge in the presence of gravity force is not symmetrical about the r -axis, Equation 8 could not be used as a boundary condition. Therefore, boundary conditions for Equation 11 are only given at solid contact point. To solve YLE in the presence of gravity (Equation 11) with the corresponding boundary conditions, variable substitution method is used. In this method, the second order non-linear ODE is transformed into two first-order ODEs. Afterwards, this set of ODEs can be solved by a numerical method such as fourth-order Runge-Kutta's method.

2.2. Liquid bridge between equal spheres

To solve YLE for liquid bridge formed between two plates, either neck radius or contact radius should be specified as boundary conditions. Hence, the determination of critical fracture aperture (or critical separation distance) for this system will be dependent on either neck radius or contact radius. If one assumes a liquid bridge between two spheres under conditions that liquid bridge volume relative to solid spheres volume is very small, the liquid bridge is almost similar to a bridge between two plates.

This claim will be verified by a numerical example in the next section. Furthermore, the non-dimensional analysis of YLE for the case of liquid bridge between spheres is more sensible. Therefore, some features of liquid bridge in horizontal fractures is studied by considering a liquid bridge between two spheres. For a liquid bridge between two equal spheres, the definition of dimensionless variables leads to some simplification in the numerical procedure. By introducing $r_D = \frac{r}{R}$ and $z_D = \frac{z}{R}$ as dimensionless coordinates and $H_D = \frac{P_c \times R}{2\sigma}$ as dimensionless mean curvature Equation 1 can be written as:

$$2H_D = \frac{d^2 r_D / dz_D^2}{(1 + dr_D / dz_D)^{3/2}} - \frac{1}{r_D (1 + dr_D / dz_D)^{1/2}} \quad (12)$$

Boundary conditions needed to solve the above equation may be found at contact radius as well as neck radius of liquid bridge. Non-dimensionalizing Equations 7 and 8 gives boundary conditions at the neck of the bridge. Alternatively, boundary conditions at solid contact point are given by the following equations:

$$r_D \Big|_{z_D = \frac{b}{2R}} = \sin(\alpha) \quad (13)$$

$$\frac{dr_D}{dz_D} \Big|_{z_D = \frac{b}{2R}} = \cot(\alpha + \theta) \quad (14)$$

A modified Euler method is used to find the numerical solution of Equation 12.

2.3. Fracture capillary pressure

The two-phase flow characteristics of porous media is largely dependent on the relative permeability and capillary functions of the porous media. Romm (1966), based on the experiments of flow between two parallel glass plates, proposed straight-line relative permeability and zero capillary pressure for fractures. However, extending the results of these experiments to real fractured reservoirs is disputable. Fracture capillary pressure is used to be assumed zero in earlier attempts for the numerical simulation of fractured reservoirs (Firoozabadi and Hauge, 1990). However, this assumption has been shown to be inconsistent with multi-block gravity drainage experiments (Horie et al., 1990) as well as field observations (Thomas et al., 1987; Festoy and van Golf-Racht, 1989). The assumption of zero fracture capillary pressure is appropriate when adjacent matrix blocks in a fractured reservoir are discontinuous. Horie et al. (1990) reported that zero fracture capillary pressure as well as constant fracture capillary pressure did not match the experimental observations of gravity drainage in a stack of matrix blocks. They suggested an empirical saturation-dependent capillary pressure for fracture, similar to that of a matrix. Dindoruk and Firoozabadi (1995) used a numerical simulator to analyze the influence of matrix and fracture capillary pressure on the recovery of gravity drainage in a stack of matrix blocks. They observed that as the fracture capillary pressure decreases both recovery rate and ultimate recovery drop. They also stated that the greatest amount of recovery occurs when capillary pressure contrast between matrix and fracture is the least. Hence, they concluded that fracture capillary pressure is a driving force in the fracture, and it has the most significant influence on gravity drainage performance. De la Porte et al. (2005) examined the amount of error imposed by selecting straight-line fracture relative permeability and zero fracture capillary pressure in the numerical simulation of fractured reservoirs. They concluded that in gas-oil systems, when fracture aperture is less than 100 micron, non-zero fracture capillary

pressure should be used, especially for the reservoirs of small height matrix blocks. As an alternative for zero fracture capillary pressure, a constant value fracture capillary pressure and a saturation-dependent fracture capillary pressure may be used. Parallel plate model, which corresponds to fluid flow along the fracture surface, is a simplified form of YLE. This model describes fracture capillary pressure as a function of fracture aperture and surface wettability, as expressed by:

$$P_{cf} = \frac{2\sigma \cos\theta}{b} \quad (15)$$

The above equation is derived based on the assumption that one of the principal radii of curvature is infinity ($R_2 = \infty$) and the other one is $R_1 = \frac{b}{2\cos\theta}$. This equation leads to a constant fracture capillary pressure for a specific fracture. In other words, fracture capillary pressure predicted by Equation 15 is not dependent on liquid volume (or liquid saturation) in the fracture. On the other hand, several researchers suggested that the capillary pressure of fracture may be considered saturation-dependent, similar to the matrix capillary pressure. Reitsma and Kueper (1994) performed experimental measurements of oil-water fracture capillary pressure for a rough-walled fracture. They used models of Brooks and Corey (1964) and Van Genaukten (1980) to match experimental results. Dindoruk and Firoozabadi (1995) proposed a saturation-dependent model to represent fracture capillary pressure. This model assumes that the shape of fracture capillary pressure curve is similar to that of a matrix although the curvature is mainly different. Equations of Brooks and Corey (1964), Van Genaukten (1980), and Dindoruk and Firoozabadi (1995) for the models of fracture capillary pressure are tabulated in Table 1.

Table 1
Available models to predict fracture capillary pressure.

Author	Equation	Definitions
Brooks-Corey	$P_{cf} = P_d (S_e)^{\frac{1}{\lambda}}$	$S_e = \frac{S_{of} - S_{orf}}{1 - S_{orf}}$
van Genuchten	$P_{cf} = P_o \left(S_e^{\frac{1}{m}} - 1 \right)^{\frac{1}{n}}$	$S_e = \frac{S_{of} - S_{orf}}{S_{osf} - S_{orf}}$
Dindoruk and Firoozabadi	$P_{cf} = P_{cf}^o - \sigma_f \left \ln \left(\frac{S_o - S_{orf}}{1 - S_{orf}} \right) \right ^{n_f}$	$P_{cf} > P_{cf}^o$

The equations presented in Table 1 are able to match the available experimental measurements of fracture capillary pressure as a function of fracture saturation as well as gravity drainage experiments in stacked blocks. However, the theoretical basis of such interdependency of fracture capillary pressure and saturation needs to be addressed.

3. Experiments

An experimental setup was designed to observe and measure the characteristics of liquid bridges, as shown in Figure 2. This setup is mainly made up of two plates which are attached to a main frame. Two glass slabs are glued to the plates; glass slabs act as a substrate for the creation of liquid bridge. The lower plate is stagnant, but the upper one is movable; it can make a vertical translation with an accuracy of 4 micrometers. In order to create a liquid bridge, a drop with a specified volume is placed on the lower plate by using a pipette. Then, the upper plate is moved toward the drop, and a liquid bridge is formed. After

the creation of liquid bridge, the upper plate is moved upward in several steps, and the images of liquid bridge are recorded at each step. This process continues until the breakage of bridge. Three different values for the volume of the deposited droplets used to create liquid bridge are 5, 7.5, and 10 μL .

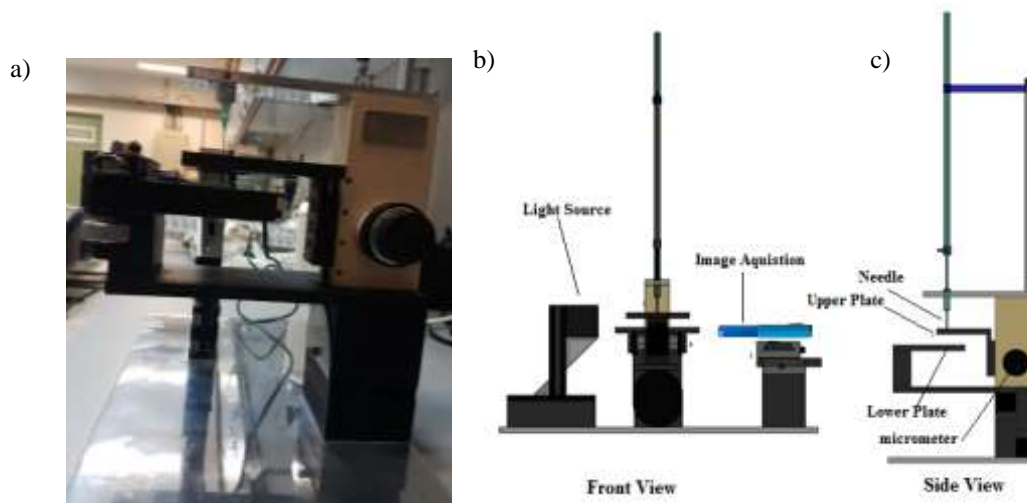


Figure 2

a) experimental setup, b) a schematic figure (front view), and c) a schematic figure (side view).

4. Results and discussion

4.1. Experiments

As stated in the previous section, the images of liquid bridge were recorded by a camera. Figure 3 illustrates liquid bridges at successive steps for 5, 7.5, and 10 μL bridges. As seen in Figure 3, from left to right, the distance between two glass substrates increases until the breakage of liquid bridge.

The maximum fracture aperture that could maintain a liquid bridge stable is called critical fracture aperture. The critical fracture aperture for 5, 7.5, and 10 microliters was measured to be 2.1, 2.37, and 2.5 mm respectively. Contact angle, liquid bridge contact radius, and liquid bridge neck radius for liquid bridges at different volumes and at different apertures were computed by using IMAGEj software. The obtained results for the selected images are presented in Table 2. It is observed that contact angle varies between 20 and 45 degrees, but a value about 30 to 35 degrees is the most common value of contact angle in the solid-fluid system used in this study.

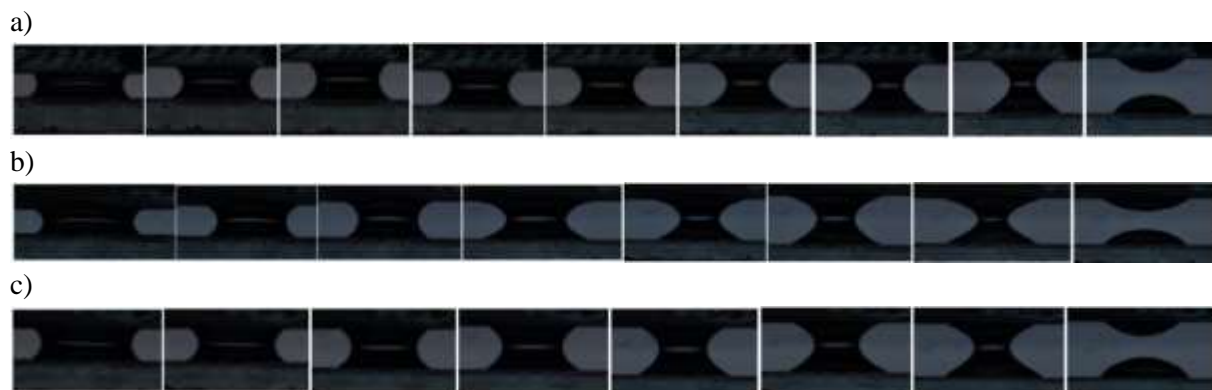


Figure 3

Sequential images of a) 5 μL , b) 7.5 μL , and c) 10 μL liquid bridges.

Table 2
Measured contact angles of liquid bridges.

Volume	b (mm)	Contact angle	Upper contact radius (mm)	Lower contact radius (mm)	Neck radius (mm)
5 microliters	1.915	38.4	1.14	1.331	0.529
	2.051	43.5	1.139	1.317	0.449
	0.72	20.1	1.45	1.55	1.265
	1.25	31.8	1.198	1.313	0.884
	0.757	30.5	2.129	2.043	1.831
7.5 microliters	1.292	30.6	1.574	1.511	1.103
	1.877	31.7	1.559	1.463	0.772
	2.322	39.6	1.561	1.5	0.482
10 microliters	1.409	31.9	1.529	1.596	1.132
	1.961	35.4	1.392	1.544	0.815

4.2. Numerical solution of YLE

To obtain the numerical solution of YLE in the absence of gravity (Equation 1) and in the presence of gravity (Equation 11), boundary conditions must be specified. The results of the image analysis of liquid bridges, as given in Figure 3, were used to determine boundary conditions. To check the validity of the numerical method, both solutions (solution I and II) are compared with the corresponding experimental results for the liquid bridge volumes of 5, 7.5, and 10 microliters in Figures 4-6 respectively. As can be seen, both solutions give a reasonable match; however, as expected, the solution in the presence of gravity shows a more perfect match.

Thereafter, the mean curvature of meniscus computed from the numerical solution is used to calculate the corresponding capillary pressure. In the absence of gravitational effects, meniscus mean curvature is constant along the liquid bridge, so capillary pressure is the same throughout the bridge. On the other hand, when gravity is included, mean curvature and capillary pressure vary across the bridge. As shown in Figures 7-9, as going upward through the bridge, capillary pressure increases. Hence, the maximum capillary pressure occurs at the upper end of bridge, i.e. the lower face of the upper matrix block. The amount of capillary pressure difference across the fracture due to gravity depends on density difference between gas and liquid phases. As can be seen in Figures 7-9, the difference in capillary pressure between the top and bottom of liquid bridge in the presence of gravity is less than 30 Pascal (0.004 psi) for all the cases, which is insignificant relative to the typical values of capillary pressure of the matrix. Furthermore, the values of fracture aperture in fractured reservoirs are usually one to two order of magnitudes smaller than the values of fracture aperture implemented in our experiments. As a consequence, for liquid bridges created in fractured reservoirs, the values of Bond number (as defined by Equation 10) is expected to be less than the values of the experiment. Therefore, it is expected that the role of gravity on liquid bridge characteristics in reservoir conditions is trivial and could be ignored.

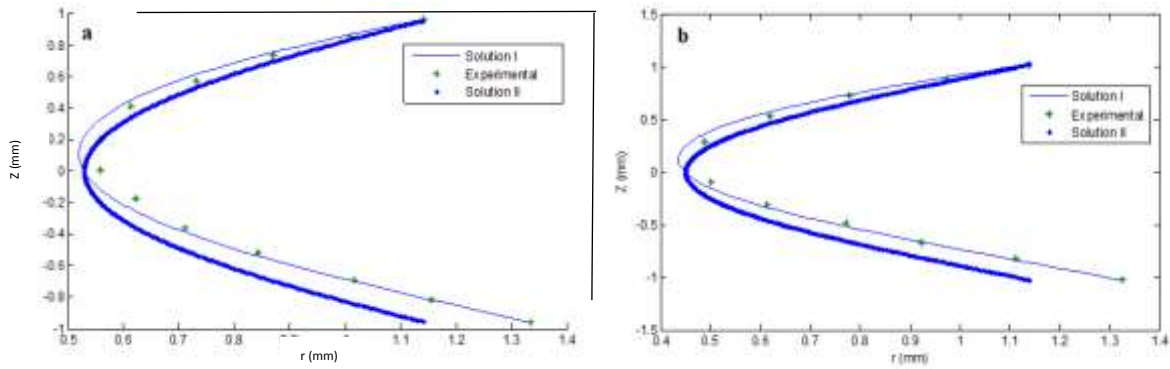


Figure 4

Meniscus profile of 5 microliters bridge computed in the presence (Solution I) and in the absence (Solution II) of gravity compared with the experimentally measured profile at fracture apertures of a) 1.915 mm and b) 2.051 mm.

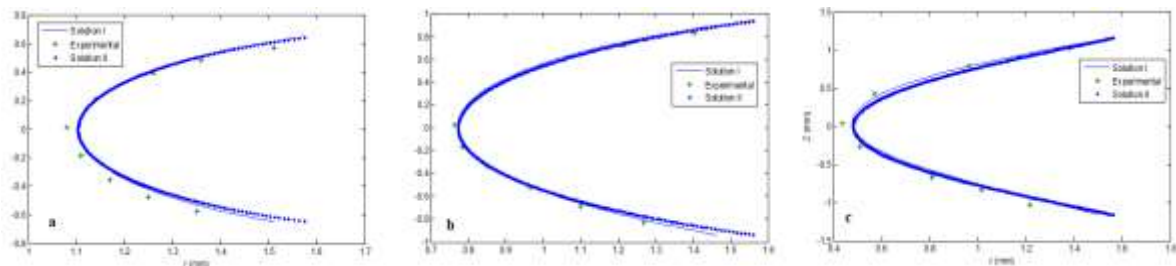


Figure 5

Meniscus profile of 7.5 microliters bridge computed in the presence (Solution I) and in the absence (Solution II) of gravity compared with the experimentally measured profile at fracture apertures of a) 1.292 mm, b) 1.877 mm, and c) 2.322 mm.

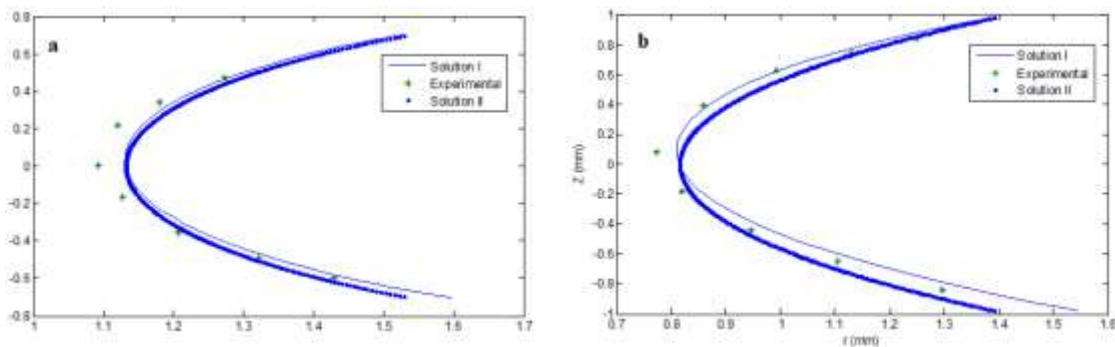


Figure 6

Meniscus profile of 10 microliters bridge computed in the presence (Solution I) and in the absence (Solution II) of gravity compared with the experimentally measured profile at fracture apertures of a) 1.409 mm and b) 1.961 mm.

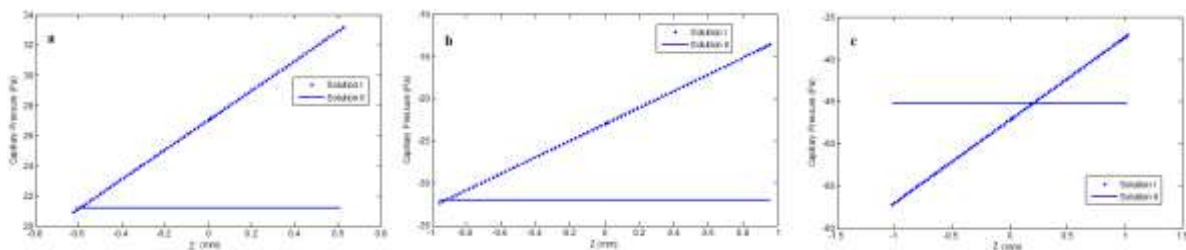


Figure 7

Variation of the capillary pressure along the free surface for a 5 microliters liquid bridge at different fracture apertures; a) 1.25 mm, b) 1.915 mm, and c) 2.051 mm.

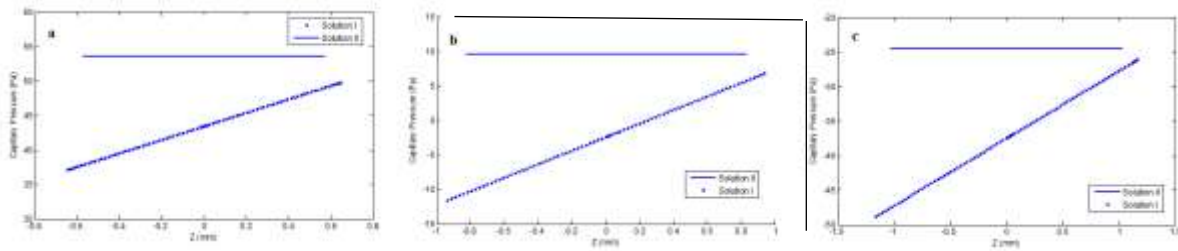


Figure 8

Variation of the capillary pressure along the free surface for a 7.5 microliters liquid bridge at different fracture apertures; a) 1.292 mm, b) 1.877 mm, and c) 2.322 mm.

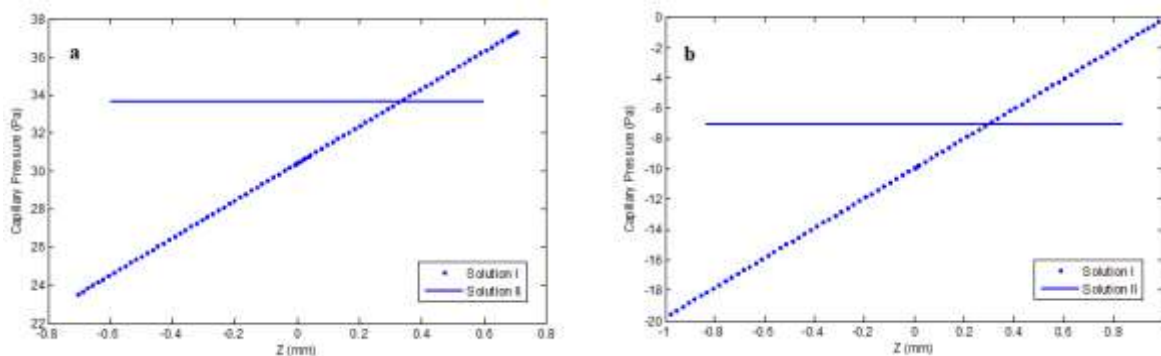


Figure 9

Variation of the capillary pressure along the free surface for a 10 microliters liquid bridge at different fracture apertures; a) 1.409 mm and b) 1.961 mm.

A clear observation from Figures 7-9 is that for a liquid bridge with a constant volume, by increasing fracture aperture, fracture capillary pressure decreases, and for some thick fractures, fracture capillary pressure has negative values. The so-called parallel plate method, as expressed in section 2.3, predicts a similar trend for capillary pressure; however, this model could not predict negative values of fracture capillary pressure at any apertures. Parallel plate method always predicts the positive values of capillary pressure at contact angles smaller than 90 degrees. It should be mentioned that fracture apertures corresponding to negative fracture capillary pressures are considerably greater than the typical values of fracture apertures in naturally fractured reservoirs. Therefore, it is expected that liquid bridges with negative capillary pressures are not probable in reservoir conditions.

4.3. Liquid bridge stability analysis

Lian et al. (1993) pointed out that for a liquid bridge between two equal spheres with a specified volume and separation, there are two solutions that satisfy YLE. The solution that corresponds to lower free surface energy is the stable solution, and the other one is the unstable solution. As the separation distance between spheres increases up to a critical value, both of the solutions converge to a unique solution. For separation distances greater than this critical value, no solution exists. This critical value is considered to be the rupture distance of liquid bridge, which is called critical fracture aperture in this study. Lian et al. (1993) used these criteria to find out the rupture distance of liquid bridge. The aforementioned procedure to find the critical separation distance for liquid bridges between equal spheres is reported to be consistent with experimental results obtained by Mason and Clark (1965).

As mentioned before, liquid bridge between two equal spheres mimics the liquid bridge between two plates provided that liquid bridge volume to sphere volume is small enough. To validate this idea, the gas-liquid interface of liquid bridge was computed for both cases, i.e. liquid bridge between spheres and liquid bridge between plates, at different liquid bridge volumes. By defining dimensionless bridge volume as the ratio of liquid bridge volume to the volume of solid sphere, it was discovered that for dimensionless volumes equal or smaller than 0.0003, both solutions predict a similar bridge. Figure 10 shows the liquid bridge interface profile between two plates and between two spheres computed by the numerical solution at two different fracture apertures and a dimensionless volume of 0.0003. It is obvious that the match between the two solutions is almost perfect. As a consequence, a liquid bridge between two equal spheres at a dimensionless volume of 0.0003 was considered to determine the critical fracture aperture.

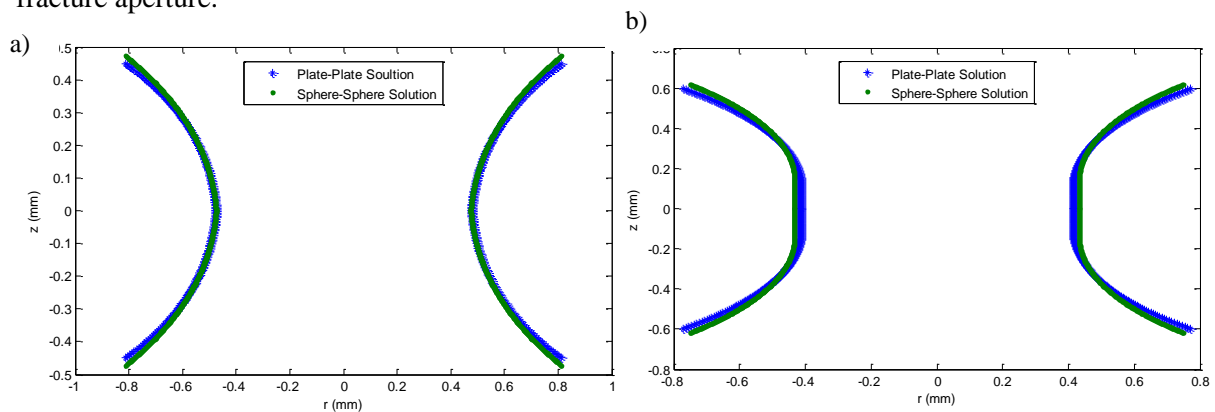


Figure 10

Comparison of the gas-liquid interface profile of liquid bridge between two plates with the profile of liquid bridge between two spheres plates at fracture apertures of a) 0.9 mm and b) 1.2 mm; contact angle=30 degrees and dimensionless volume=0.0003.

The critical fracture aperture was determined as a function of liquid bridge volume at different contact angles by the procedure suggested by Lian et al. (1993). The calculations were made based on a liquid bridge between two spheres at a dimensionless volume of 0.0003. At first, the dimensionless critical distance (or critical fracture aperture) is computed at each contact angle. Afterwards, non-dimensional values are converted to dimensional critical apertures, considering different liquid bridge volumes. The computed values of the critical fracture aperture as well as experimentally measured values are shown in Figure 11. The critical fracture aperture measured from the analysis of the images captured during the experiments illustrated in section 3 are also compared with the numerically computed values in Figure 11. The location of the experimentally measured values of the critical fracture aperture in Figure 11 represents that contact angles varies between 15 to 30 degrees; however, based on liquid bridge shape analysis in section 4.1, contact angles lie between 20 to 40 degrees. This small discrepancy between the experimental and numerical results may be attributed to gravity effects or experimental errors.

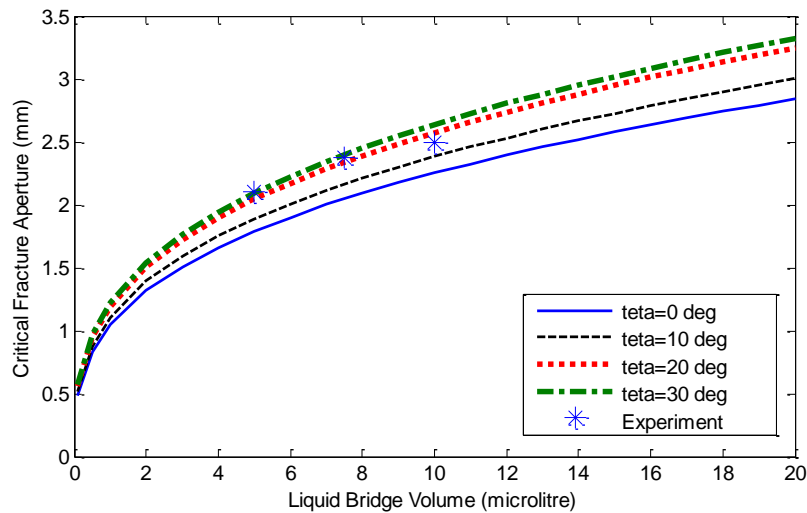


Figure 11

Comparison of the experimentally measured critical fracture apertures as a function of liquid bridge volume with the numerically calculated values at contact angles of 0, 10, 20, and 30 degrees.

It can be inferred from Figure 11 that the critical fracture aperture is an increasing function of both liquid bridge volume and solid-liquid contact angle. Accordingly, to develop an empirical relation that correlates the critical fracture aperture to the contact angle and bridge volume, a non-linear regression analysis was conducted on the available dataset of the numerical solution. A functional form similar to the one proposed by Lian et al. (1993) was used as follows:

$$b_c = a \sqrt[b]{V} (c + d \theta) \quad (16)$$

where, a , b , c , and d are regression coefficients. The values of these coefficients based on the best regression fit ($R^2 = 0.999$) are presented in Table 3. The obtained fit is compared with the data in Figure 12. As can be seen in Figure 12, the obtained correlation matched the numerical solution results very well. The proposed correlation is able to predict the critical fracture aperture for liquid bridges in horizontal smooth fractures. For the case of inclined fractures, the abovementioned correlation is applicable provided that gravitational effects are insignificant. It is also worth mentioning that the properties of liquid- and gas-like surface tension and density has no effect on liquid bridge stability (and therefore on the critical fracture aperture) in the absence of gravity. However, if surface tension is such low that miscibility occurs, variations in liquid bridge volume due to mass transfer may change fracture capillary pressure.

Table 3
Coefficients of the regression fit.

a	0.8147
b	3
c	1.29
D	0.4277

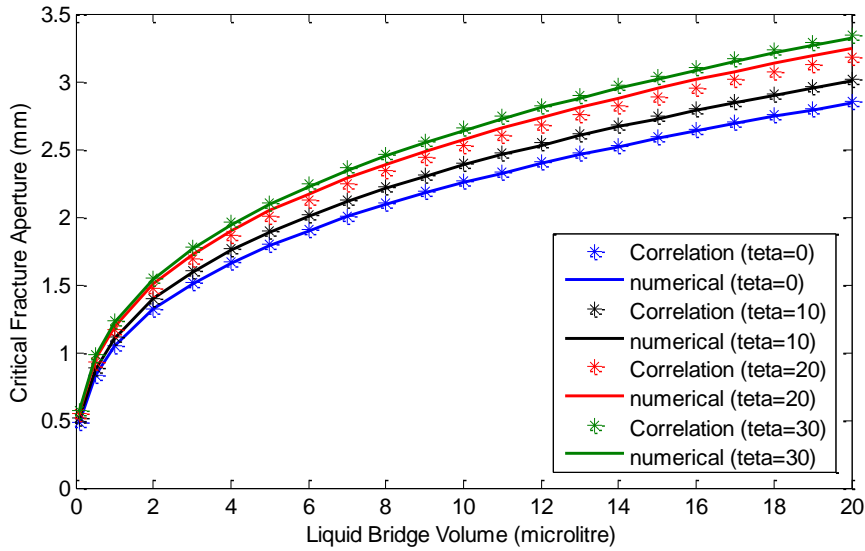
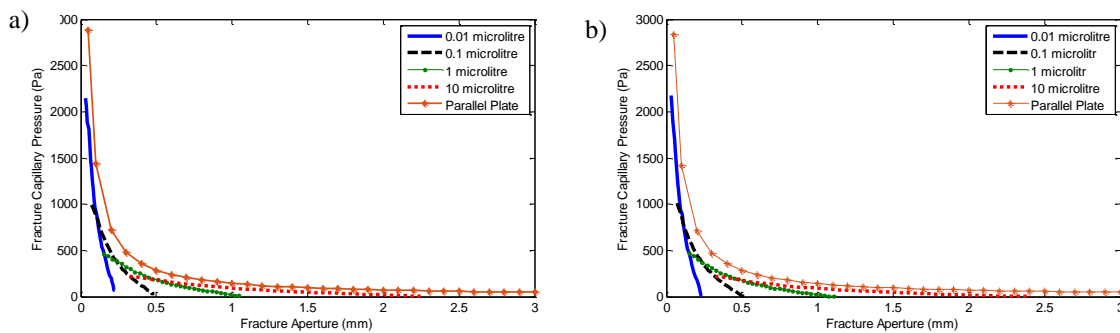


Figure 12

A comparison of the numerically computed critical fracture aperture as a function of liquid bridge volume with the values calculated by the proposed correlation at different contact angles.

4.4. Analysis of fracture capillary pressure

The intention of this section is to relate fracture capillary pressure to liquid bridge volume, fracture aperture, and contact angle. To this end, the dimensionless mean curvature of a liquid bridge at a given contact angle and dimensionless aperture is first calculated based on the procedure depicted in section 2.2. Then, capillary pressure is computed at various liquid bridge volumes. Since air and water were the fluid system utilized in our experiments, the properties of air and water were used to compute fracture capillary pressure in this section. The surface tension of air-water system is 72 dyne/cm, whereas oil-gas surface tension in reservoir conditions is usually less than 1 dynes/cm. By multiplying the calculated values of capillary pressure by the ratio of actual surface tension (in dynes/cm) to 72, capillary pressure in reservoir conditions could be obtained. Capillary pressure is calculated by solving the exact form of YLE (Equation 12) as well as the simplified form of YLE (Equation 15), i.e. parallel plated model, as a function of fracture aperture at different contact angles. The obtained results are given in Figure 13.



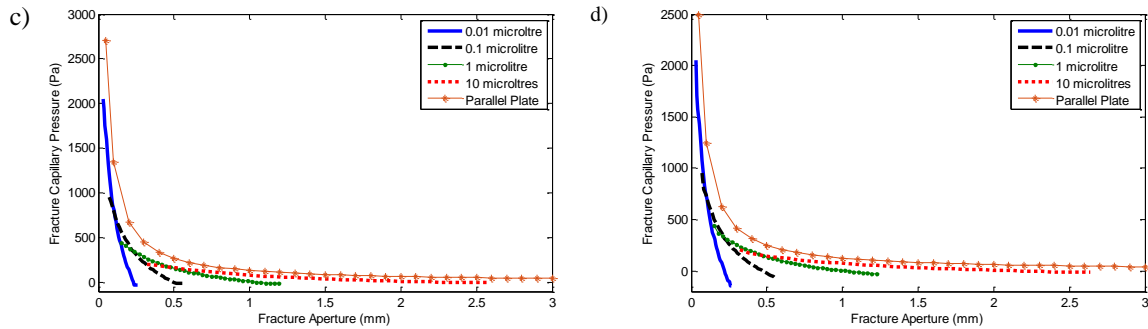


Figure 13

Fracture capillary pressure versus fracture aperture at liquid bridge volumes of 0.01, 0.1, 1.0, and 10 microliters at contact angles of a) zero, b) 10, c) 20, and d) 30 degrees; calculations are based on a dimensionless liquid bridge volume of 0.0003.

As can be seen, capillary pressure predicted by parallel plate method (Equation 15) is independent of liquid bridge volume and is only a function of fracture aperture and contact angle. Therefore, parallel plate model predicts a constant value for capillary pressure at any liquid saturation in the fracture. However, the trend of the variation of capillary pressure with fracture aperture predicted by both solutions is similar. On the other hand, the numerical solution of the exact form of YLE reveals that fracture capillary pressure is a decreasing function of liquid bridge volume. When liquid bridge volume is sufficiently small (say less than 0.5 microliters), which corresponds to low liquid saturation in fracture, fracture capillary may reach values more than 0.1 psi, depending on fracture aperture. Another weakness of parallel plate method is that this model does not consider the stability of liquid bridge. However, parallel plate method predicts capillary pressure at any fracture apertures regardless of whether liquid bridge is stable or not. Hence, parallel plate method is not a good predictive model for capillary pressure in a range of liquid saturations in the fracture.

Figure 14 represents fracture capillary pressure as a function liquid bridge volume at different fracture apertures and different contact angles. The illustration of fracture capillary pressure as a function of liquid bridge volume is helpful in the clarification of the shape of P_{cf} -Saturation curve. During gas-liquid gravity drainage in fractured porous media, as the fracture liquid desaturates, the liquid saturation of fracture decreases. Horie et al. (1990) believed that the fracture may have a very low irreducible wetting-phase saturation compared with the matrix. Firoozabadi and Markeset (1994), based on the visual inspection of gas-oil gravity drainage experiments in multi-block systems, reported that the formation of liquid bridges only occurs until a certain time in gravity drainage process. They observed that this time is related to the fracture aperture between matrix blocks, and after this time, the process of formation and breakdown of liquid bridge in fracture stops or is very slow. It means that at later times of gravity drainage that gas saturation in the fracture is increased, low amounts of liquid exit in fracture; therefore, the critical fracture aperture to stabilize liquid bridges with such small volumes is very low. In this case, if fracture is not narrow enough, no stable liquid bridge is formed. As a consequence, considerable values of fracture capillary pressure can be realized provided that both fracture liquid saturation and fracture aperture are sufficiently small. It is worth mentioning that although low values of bridge volume may yield high fracture capillary pressures, the fracture aperture should be small enough to stabilize such small liquid bridges. In other words, the critical fracture aperture for smaller liquid bridges is lesser; therefore, fracture aperture which are sufficiently narrow could retain such small liquid bridges. Consequently, to have significant fracture capillary pressure, two conditions must be satisfied: small size liquid bridge (or low liquid saturation in fracture) and thin fractures (low fracture apertures).

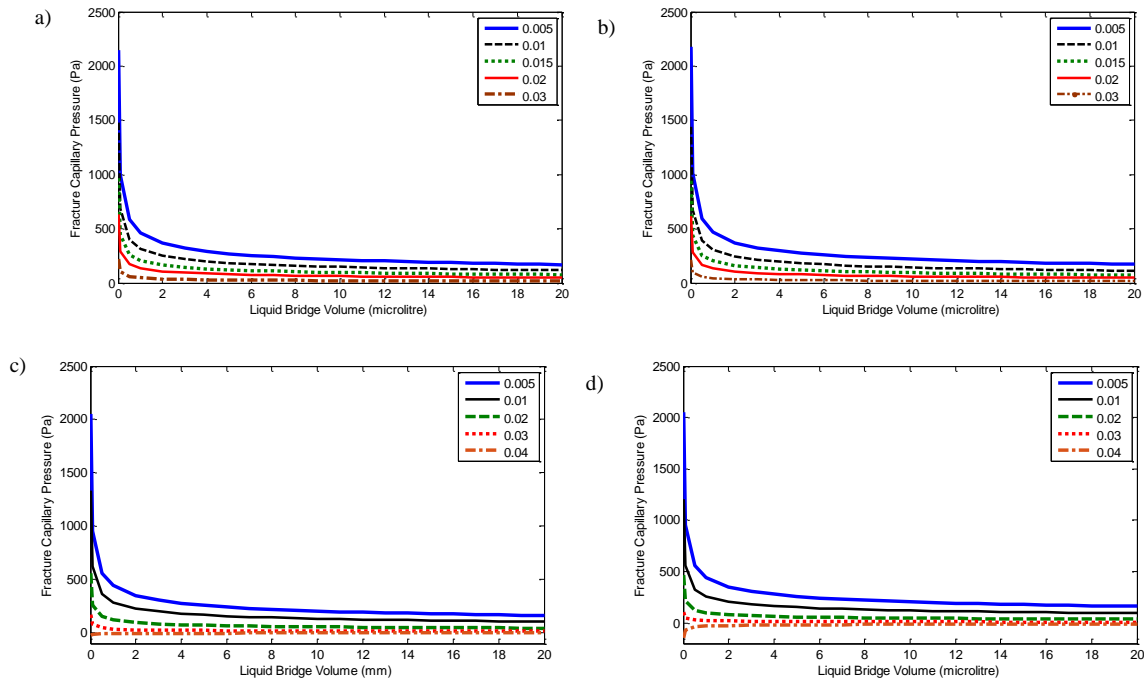


Figure 14

Fracture capillary pressure versus liquid bridge volume at different values of dimensionless half aperture and at contact angles of a) zero, b) 10, c) 20, and d) 30 degrees; calculations are based on a dimensionless liquid bridge volume of 0.0003.

One of the features of P_{cf} -Volume curves, as shown in Figure 14, is that fracture capillary pressure varies sharply with volume at low liquid bridge volumes, which is similar to the variation of a porous medium with saturation. It can be seen that the shape of P_{cf} -Volume curve is similar to that of a typical porous media. Liquid bridge volume is representative of liquid saturation in fracture, so it is expected that P_{cf} -Volume and P_{cf} -Saturation curves have a similar shape. The curves illustrated in Figure 14 justifies the use of porous-medium type capillary pressure functions for fracture as suggested by Dindoruk and Firoozabadi (1995). There are some exceptions for dimensionless half apertures greater than 0.003, in which capillary pressure has negative values; however, it is expected that in real fractured reservoir conditions, where fracture apertures are much smaller, negative capillary pressures do not occur.

A careful view of Figures 13-d and 14-d reveals that as the fracture thickness reaches its maximum (critical) aperture, capillary pressure may have negative values. Moreover, computing capillary pressure for liquid bridges in the experiments showed that negative capillary pressures exist, especially for greater fracture apertures. It is worth mentioning that since negative capillary pressure occurs in thick fractures, highly negative values are not possible. In fact, for large fracture apertures corresponding to highly negative values of capillary pressure usually no stable liquid bridges exist. In other words, fracture apertures which correspond to negative capillary pressures are almost near the critical fracture aperture; therefore, only small negative values are realized. Theoretically, a negative capillary pressure is expected to reduce the liquid drainage from matrix blocks although liquid bridges with highly negative capillary pressures are not stable. Thus, for thick fractures, liquid bridges either do not exist or are not efficient to maintain capillary continuity. The existence of liquid bridges with negative capillary pressures was also observed by Lian et al. (1993) in the study of liquid bridges between two spheres.

5. Conclusions

In this study, different characteristics of liquid bridges in horizontal fractures are investigated through mathematical and experimental methods. The main findings of this study may be summarized as follows:

1. The numerical solutions of YLE equation in the presence and in the absence of gravity were compared with the experimental measurements. Both solutions showed a reasonable match, but the solution in the presence of gravity better matched the experimental data.
2. It was shown that the solution of YLE for the liquid bridge between spheres is almost the same as the liquid bridge between plates provided that the ratio of liquid bridge volume to sphere volume is sufficiently small.
3. Liquid bridge stability was specified as a function of liquid bridge volume and contact angle. Non-linear regression analysis was performed to find an empirical relation for the prediction of the critical fracture aperture.
4. Fracture capillary pressure was computed as a function of liquid bridge volume and fracture aperture at different contact angles. It was shown that at lower volumes of liquid bridge in narrower fracture apertures, fracture capillary pressure may reach values comparable with matrix capillary pressure. Furthermore, the shape of P_{cf} versus liquid volume (a representative of liquid saturation in fracture) curve was observed to be similar to that of a matrix, which justifies the utilization of the capillary pressure-saturation functions of porous media for fracture media.

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